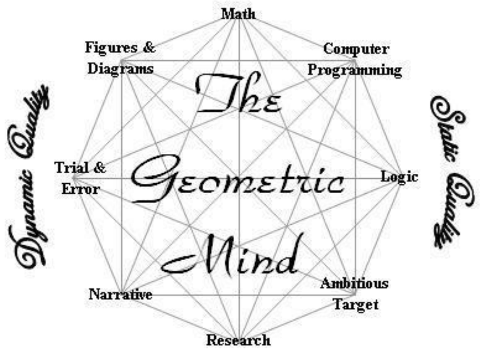
THE GEOMETRIC MIND SERIES
an *auto*SOCRATIC QUICK-START publication

Home Sweet Home

A Short Story on Mortgage Payments and Interest Rates



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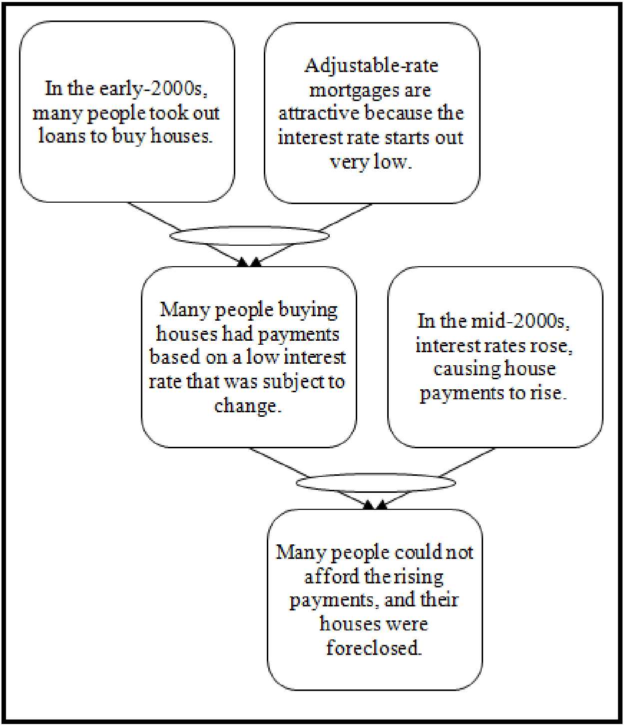
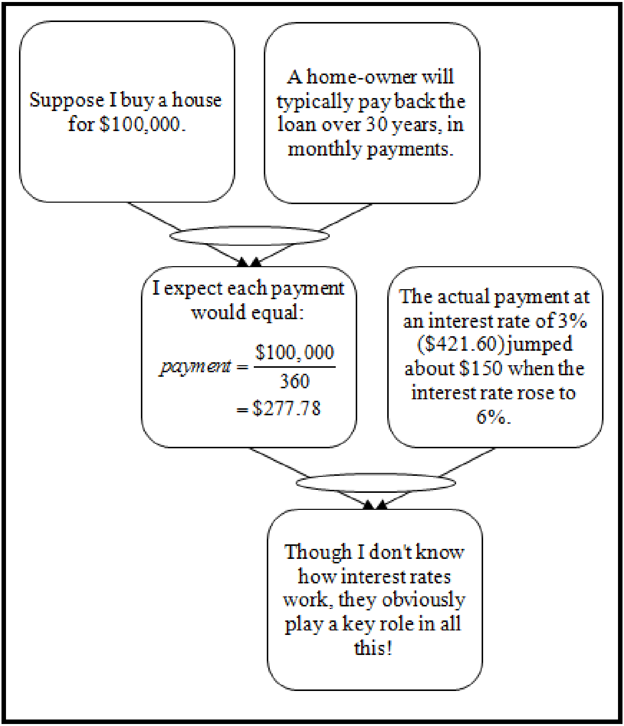
The Housing Crisis

What caused payments to go up?



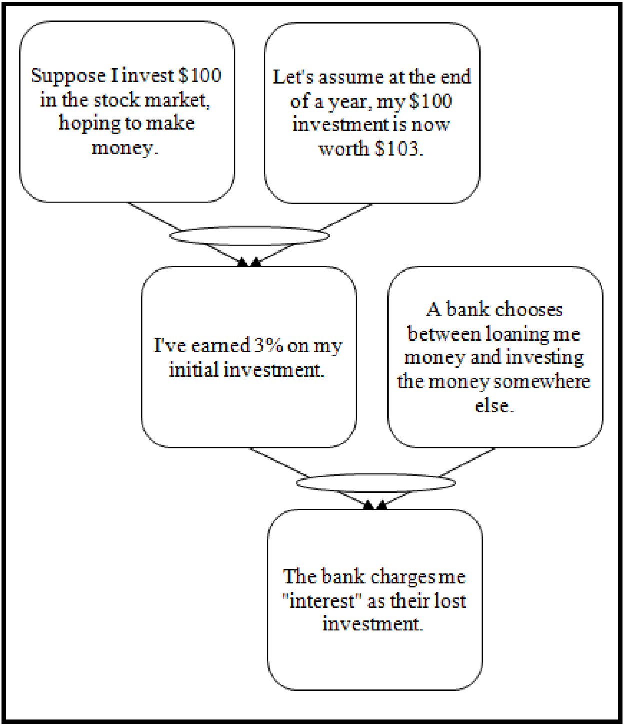
House Payment Calculation

How are house payments calculated?



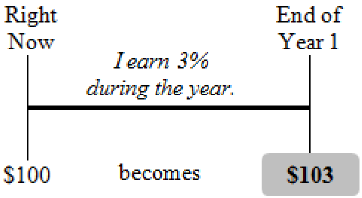
Interest Rates

What are they and how do they work?



Rates of Return

Looking at Just the First Year



Another way to write this is:

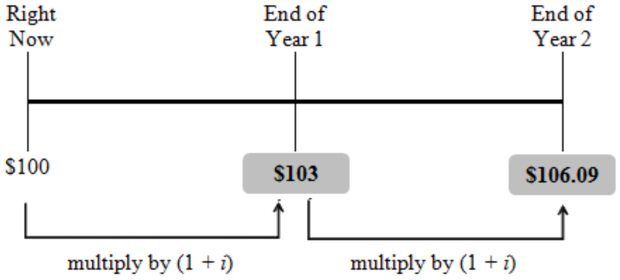
(starting money)(1 + i) = ending money

(100)(1+.03) = 103

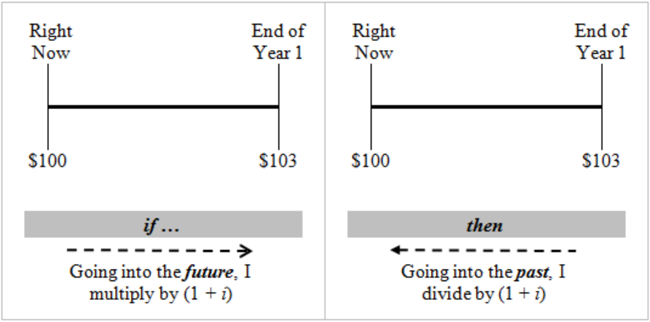
What if I earned 3% on the money in the *second* year? How much would I have?

Interest Rates

Looking at the Second Year



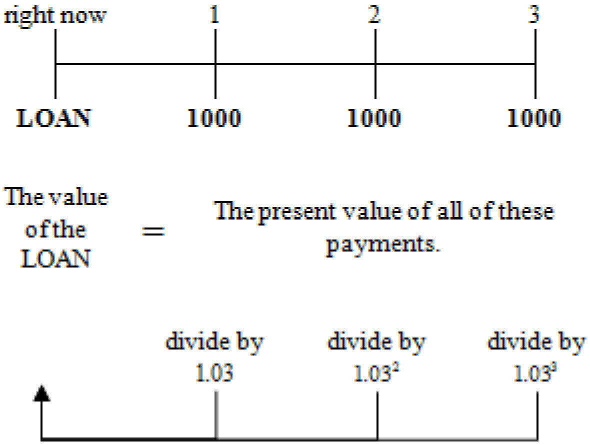
Into the FUTURE and Into the PAST



House Payments

How Does This Help Me With House Payments?

Let's start with a simple example: you make 3 payments of $1,000 at the end of each year, with an interest rate of 3%. Let's diagram this:



Actually doing the calculation, I have:

$$\text{LOAN} = \frac{1000}{1.03} + \frac{1000}{1.03^2} + \frac{1000}{1.03^3}$$

$$= 970.87 + 942.60 + 915.14$$

$$= 2828.61$$

Therefore: a loan of $2,828.61 at 3% is equal to three payments of $1,000 each.

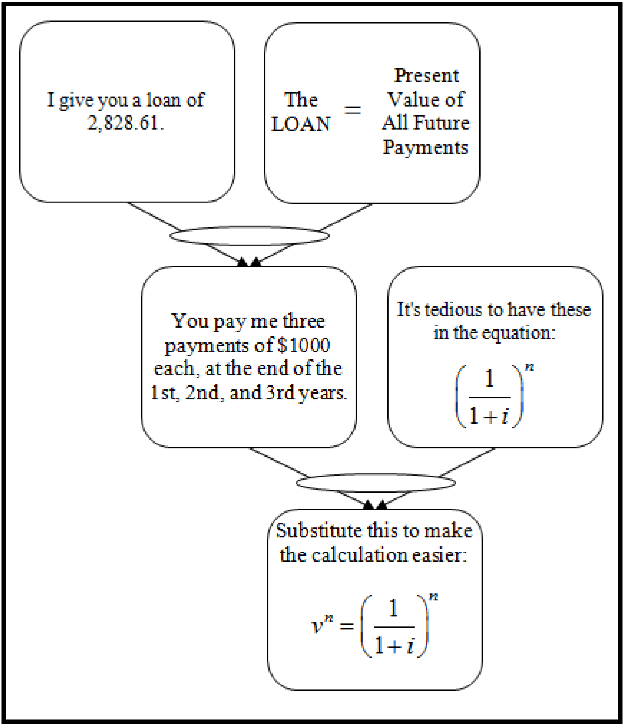
THE TIMELINE

IS THE KEY

The key is to bring all payments back to the loan period. BUT YOU ACTUALLY HAVE TO DRAW THE TIMELINE!

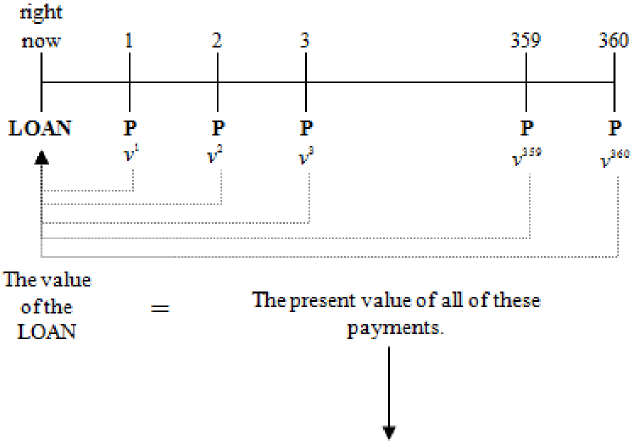
A Sample Calculation

And some easier notation



The Home Loan and My Payments

The Two are Equal!

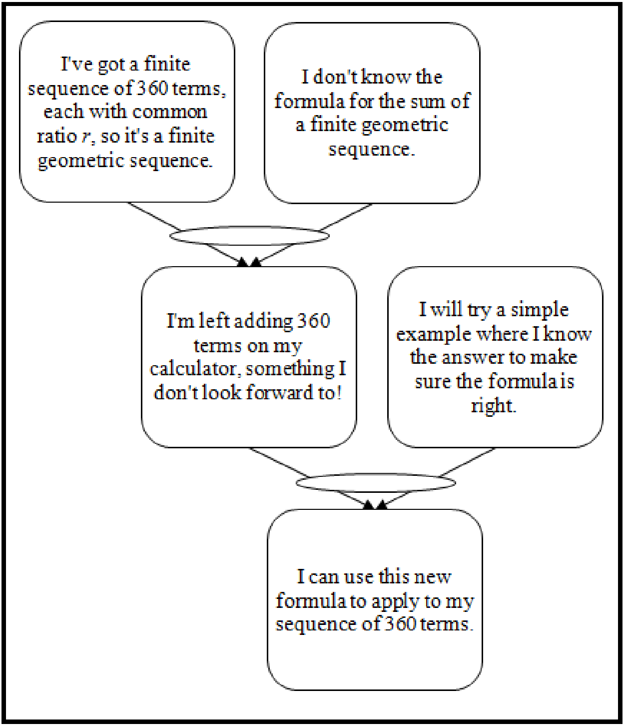


$$\text{LOAN} = Pv^1 + Pv^2 + Pv^3 + \cdots Pv^{359} + Pv^{360}$$

The previous example of three $1,000 payments required only three calculations. Above is 360 calculations! There must be an easier way!

A Finite Geometric Sequence

Finding the Sum of Many Terms



A Finite Geometric Sequence

Finding the Sum of a Lot of Terms

$$S = 1 + 2 + 4 + 8 + 16$$

What happens if I double everything? I've doubled everything, because the ratio between each term is '2'. The two equations are:

$$2S = 2 + 4 + 8 + 16 + 32$$

$$S = 1 + 2 + 4 + 8 + 16$$

Why did I do this? Because I can subtract one equation from the other, and most of the terms disappear:

$$2S = 2 + 4 + 8 + 16 + 32$$

$$S = 1 + 2 + 4 + 8 + 16$$

I subtract one entire formula from the other.

$$2S = 2 + 4 + 8 + 16 + 32$$

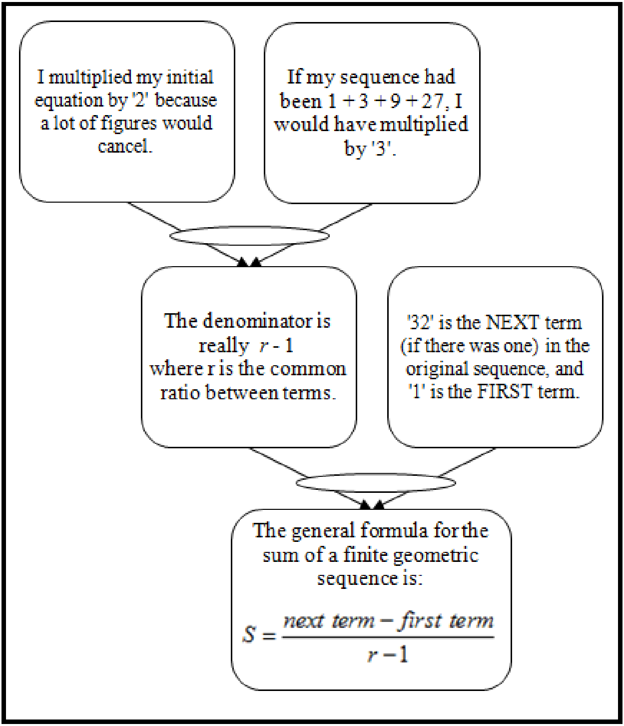
$$-\quad S = 1 + 2 + 4 + 8 + 16$$

$$2S - S = 32 - 1$$

Remember I don’t care about “32 – 1”. I’m really searching for the *general* formula for the sum of a finite geometric sequence.

A General Formula

Finite Geometric Sequence



$1 + 3 + 9 + 27$

A Finite Geometric Sequence Problem

just add them together:

use the formula for finding the sum of this finite geometric sequence

$$S = \frac{next\ term - first\ term}{r - 1}$$

1 + 5 + 25

A Finite Geometric Sequence Problem

just add them together:

use the formula for finding the sum of this finite geometric sequence

$$S = \frac{next\ term - first\ term}{r - 1}$$

PUTTING IT ALL TOGETHER

Now, I'm ready to answer my initial question, which was "find *P*":

$$\text{LOAN} = Pv^1 + Pv^2 + Pv^3 + \cdots Pv^{359} + Pv^{360}$$

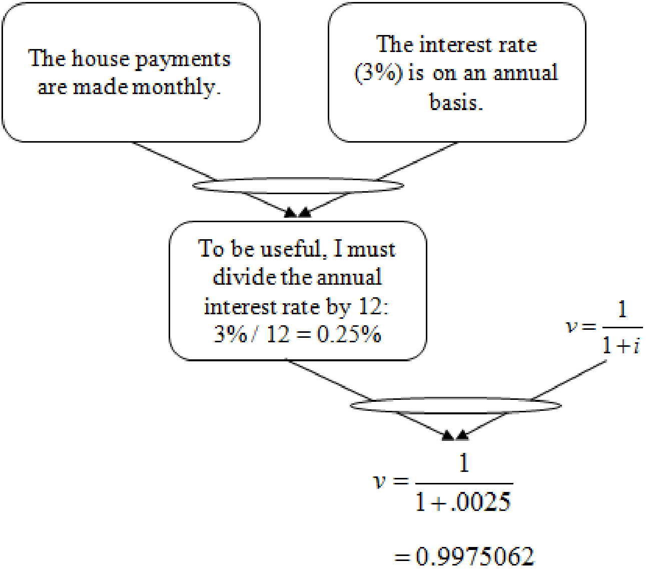
$$\text{LOAN} = Pv\left(1 + v^2 + \cdots + v^{358} + v^{359}\right) \qquad S = \frac{\textit{next term} - \textit{first term}}{r - 1}$$

$$\text{LOAN} = Pv\left(\frac{v^{360} - 1}{v - 1}\right)$$

$$\text{LOAN}\left(\frac{v - 1}{v\left(v^{360} - 1\right)}\right) = P$$

ALMOST THERE

But wait! The interest rate is an *annual* rate, but the payments are *monthly*. Let's fix that:



And, applying the specifics of my loan ($100,000 first time home-owner) with the above annual interest rate (3%) and discount factor *v*, gives me my monthly payment of $421.60.

$$\text{Loan} = 100{,}000$$
$$v = 0.9975062$$

$$\text{LOAN}\left(\frac{v-1}{v\left(v^{360}-1\right)}\right) = P$$

$$100000\left[\frac{.9975062-1}{.9975062\left(.9975062^{360}-1\right)}\right] = P$$

$$421.60 = P$$

And there it is!

THE GEOMETRIC MIND

PROBLEMS

The following three problems each have a CHECK (to make sure you've done the problem right).

Once you've confirmed you've done the problem right, there's a KEY. The key is necessary to unlock the next installment.

Key1

Key2

Key3

PROBLEM 1

I borrowed some money with the condition I would pay it back in 4 monthly installments of $150. Fortunately, there was no interest rate. How much did I borrow?

		0	.		
Key1		Check			

PROBLEM 2

Interest rates rose from 3% to 6% in the example earlier in the booklet. How much did the monthly payment rise?

6% Payment				.		
3% Payment	4	2	1	.	6	0
		7		.		
		Check	Key2			

PROBLEM 3

My home payment at 3% for 30 years was $421.60 for a $100,000 loan. I realize I can really afford to pay $500. How much can I borrow, still at 3%?

			5	9	
	Key3		Check	Check	

CONCEPT CARD

Finite Geometric Sequence

Forget the formula – write the two questions out and derive it … every time.

Loans, Present Values, etc.

Always – always – draw the timeline and the arrows back to "now".