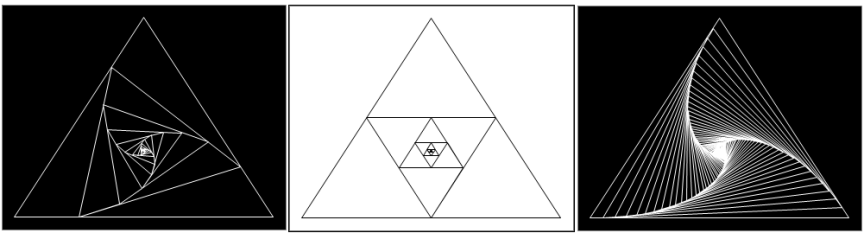
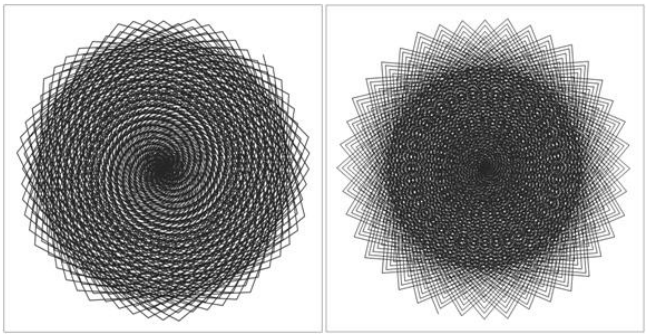
=EQUALS=

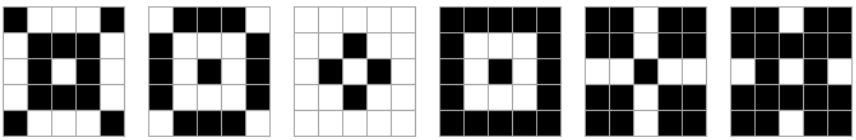
A CLUB OF INVESTIGATION AND DISCOVERY

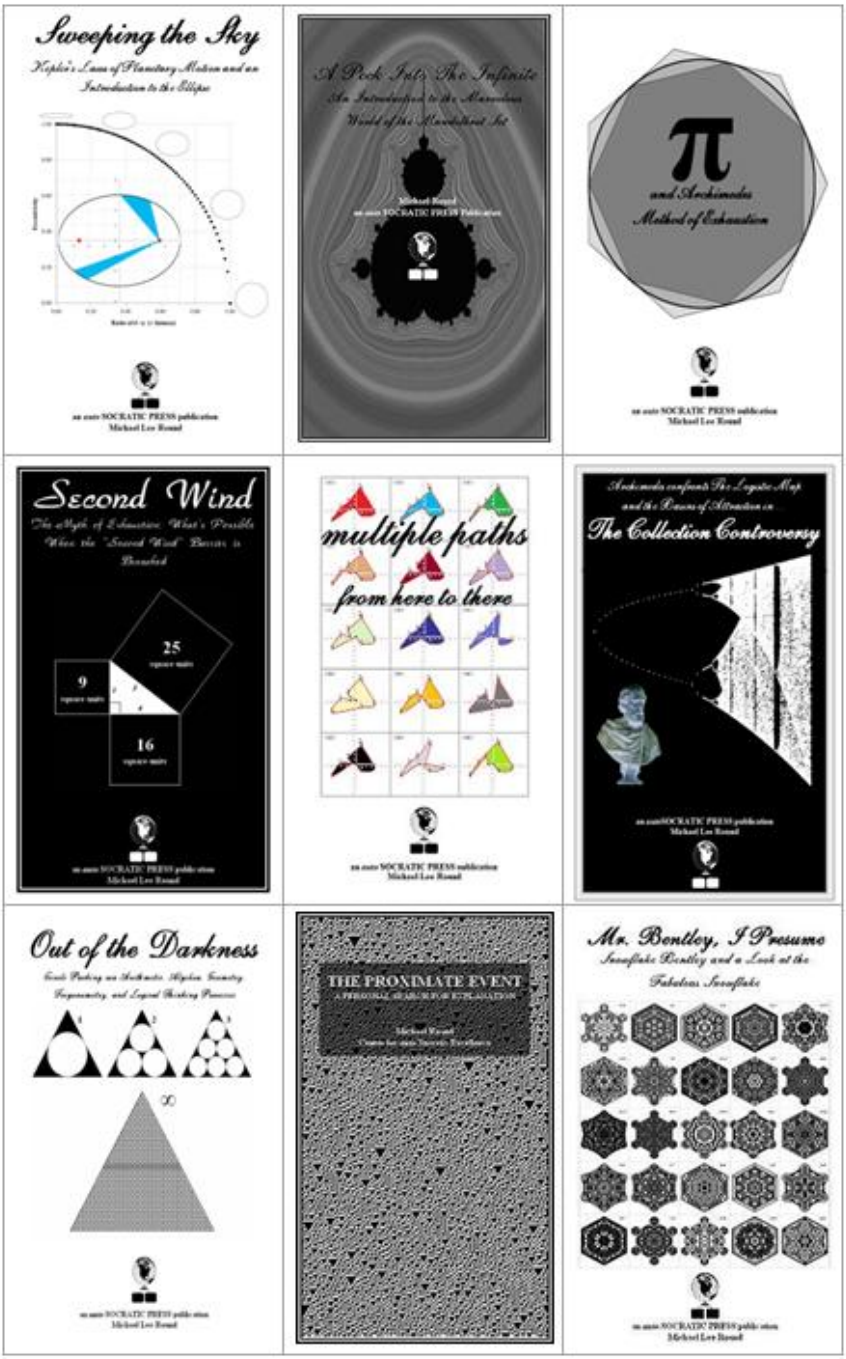
The May Agenda

Create Cool Designs
Talk about the American Statistical Association Competition
=EQUALS= This Summer





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Kepler's Laws of Planetary Motion and an Introduction to the Ellipse
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A Peek Into The Infinite
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π
and Archimedes
Method of Exhaustion
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Second Wind
The Myth of Exhaustion: What's Possible When the "Second Wind" Barrier is Banished
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multiple paths
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Archimedes confronts The Logistic Map and the Dawns of Attraction in
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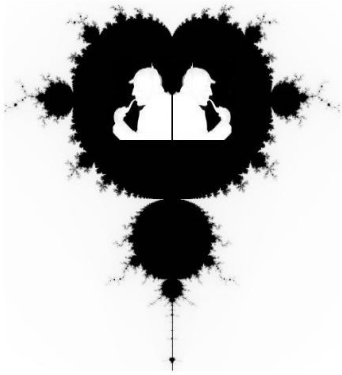

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A Club of Investigation and Discovery

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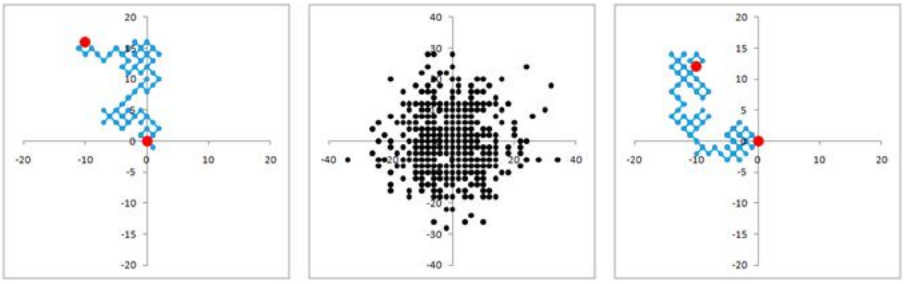
Center for *auto* SOCRATIC EXCELLENCE

THE RANDOM WALK

GOAL

To randomly move left or right, one unit, and up or down, one unit.



STEP 1: ***Calculate the First Move***

We'll start at Position (0,0). We want to move, randomly, right and left, and up and down). In columns B and C, we will use the "Random" function (that is, a random number between 0 and 1) to move. The formula is:

=if(rand()>0.5,1,-1) Type this in cells B5 and C5.

This is read as follows: IF our random number between 0 and 1 is greater than 0.5, THEN put '1' in the cell, ELSE put a '-1' in the cell.

STEP 2: ***Calculate Our New Position***

Columns D & E plot our current position after each move. What is our current for any move?

Current Position = Where we were + "Next Move".

The formula, then, for cell d5 is:

=d4+b5

Copy this formula to cell e5.

NOTE

If, at any time, you're not sure if it's calculating properly, hit the F9 KEY to recalculate!

STEP 3: ***Calculate our Move and Position for all 100 Moves***

Copy cells B4:E4, and then paste it for all 100 moves.

STEP 4: ***Graph the Data***

We've got a table of numbers, but it's hard to tell how you're moving with a table. Therefore, let's graph the data. Select all "current position" coordinates (cells d4:e104) and, using the pull-down menus, Select "Insert", and then "Scatter Plot".

NOTE

After you've created the graph, the graph is at the bottom of all the data points. I don't like that. I always highlight the graph, right click "cut", and then come back up here, and right click "paste".

NOTE 2

This is now a "Random Walk". Every time you hit the "F9" key, (recalculate), the random walk changes. There are a couple things you'll notice, and may want to change:

STEP 5: ***Seeing the Route Better***

These are just points, and you want a route. Select the data series in the graph, right click, and select "Format Data Series". Under "line color", select "solid line", and choose a color.

STEP 6: ***Fixing the Axis to See the Route Better***

The routes bounce all over the place. Why? The axis values are changing. Lets make them fixed.

Select one of the numbers on the *x*-axis, right click, and select "Format Axis".

Under "Format Options", change the "Minimum and Maximum" to "Fixed", and set these to -20 (minimum) and 20 (maximum).

Repeat the process for the *y*-axis.

STEP 7: ***Reshape the Rectangular-Looking Graph***

Our coordinates suggest a square graph, but it doesn't look square. Reshape the graph to be square.

OTHER IDEAS

What else? Here are some ideas to play around with:

IDEA 1: ***Add the Start and End Point***

This is the route of 100 random moves. We always started at (0,0), but it's hard to tell where we ended. Add a data series to show only these two points.

STEP 2: ***Write a Program to Show 100 Random Routes***

We're hitting "F9" to recalculate. Write a short macro to simulate 100 random routes.

Remember, the key to writing most macros is figuring out what you're doing — manually. Once you know that, turn on the macro-record feature and do that! To do this, Under the pull-down menu "Developer", select "Record Macro", and then do your keystrokes.

Once done, turn off the Macro-Record feature with "Developer", "Stop Recording".

Then, go back in and see what the macro recorded, clean up the macro to do what you actually want it to do, and then run it! A sample macro for this project might look like:

```
Sub Macro4()
For xxx = 1 To 100
    Calculate
Next xxx
End Sub
```

IDEA 3: *Graph the Ending Points for Many Routes*

You'd like to capture the results of the 100 random routes. Where did you finish? Write a short macro to capture these 100 ending points, and graph the results.

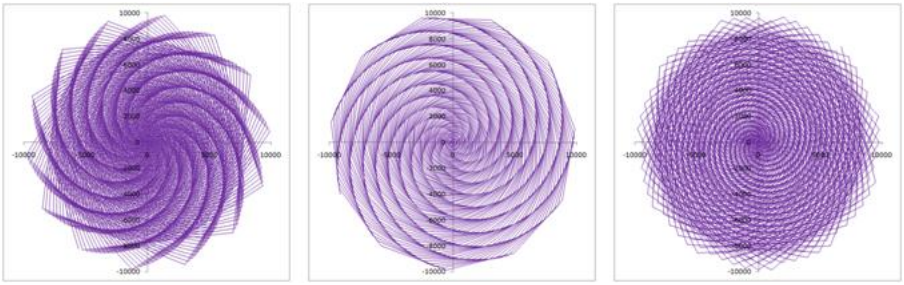
IDEA 4: *Randomize the Moves Even More*

We've restricted our movements to steps of 1 unit long. Change this to be a random distance between 0 and 2, either direction.

THE PRIME SPIRAL

GOAL

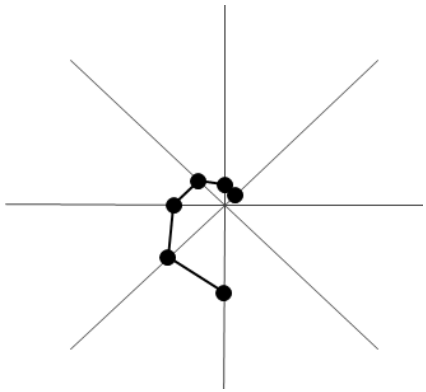
To plot prime spirals for prime numbers below 10,000



STEP 1: *We've Got the Number: We Need the Angle*

We're trying to plot the prime numbers by changing the angle. For *each* number, then, the angle must change. Let's say this change is 45 degrees, in cell b1.

Prime	Angle
2	45
3	90
5	135
7	180
11	225
13	270



Starting with angle 0, each angle must be a multiple of our "degree change" number. In cell b4, then, the formula is:

=b3+b1

If you copy that formula down, you'll see the values don't look like they should (0, 45, 90). *Why?* We no longer are referencing Cell b1. We need to "anchor" it.

In cell b4, then, the formula should be:

=b3+b1

Copy this formula to cells b4:b1231.

Now, you see each cell formula still recognizes "b1".

STEP 2: *But a Problem Arises*

We're use to working in degrees, but Excel *does not* recognize degrees – only radians! See the end of this document for more information on how to do this.

In column C, change degrees to radians with the formula:

=b3*2*3.14159/360

Copy this formula to cells c4:c1231.

STEP 3: *Finding – and Graphing – the Coordinates*

Right now, we've got a distance and an angle for each point. These are "Polar Coordinates".

We want "Cartesian Coordinates", i.e., *x* and *y* coordinates. Therefore:

in column D, the formula is: =a3*cos(c3)

in column E, the formula is: =a3*sin(c3)

Copy these formulas to cells d4:e1231.

OTHER IDEAS

What else? Here are some ideas to play around with:

IDEA 1: ***We're Entering the Degrees – and Hitting F9 – Manually. Write a Macro to do it Automatically***

We're done! But are we? This is based on a single "degree change" figure. Write a short macro to change this figure from 0 to 360, increasing by 1 degree.

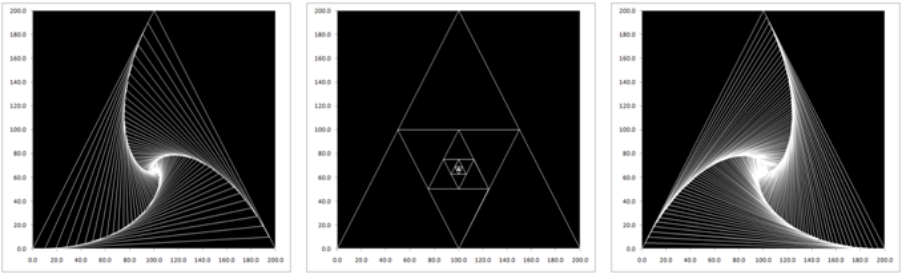
IDEA 2: ***The Prime Numbers***

Getting back to prime numbers, where did the prime numbers come from? I've put the prime numbers here. Write a program to calculate them yourself!

FUN DESIGNS

GOAL

To embed a triangle within a triangle, continually using the midpoints of each segment.



STEP 1: ***The Start of a Graph***

Create a scatter plot of the three points. You know how to do this.

Already seeing how to format the graph, reset the *x* and *y*-axis ranges to: minimum = 0, maximum = 200.

This will be a line graph, so reformat the data series to include a line.

STEP 2: ***Finishing the Triangle***

You see, with these three points, we don't have a triangle. To complete the triangle, we need our 4th point to be the same as the first point.

In cell b7, then, reference the first point with the formula

=b4

and copy this to cell c7.

STEP 3: *Constructing the Next Triangle*

For our next triangle, we need to find the midpoints of each of the corresponding line segments. What is the formula for such a midpoint?

The formula for the midpoint of two points A and B is (A + B)/2

In cell b8, then, the formula is

=(b4+b5)/2

We want to copy this to cells b8:c10.

As before, we need to close out this new triangle, by bringing the line segment back to the initial point.

In cell b11, then, reference the first point with the formula

=b8

and copy this to cell c8.

STEP 4: *Many Triangles Quickly*

You may be thinking: this is going to take a long time, entering all these formulas. You may also have noticed these last four rows all reference the cells above them.

Therefore, if we copy these four rows (cells b8:c11) into the next four open rows (select b12:c15), we'll have our next triangle!

AND SO ON!

OTHER IDEAS

What else? Here are some ideas to play around with:

IDEA 1: ***Moving Other Distances Along the Triangle***

Are we done? Maybe. This triangle figure only moves 1/2 way. What if we move 1/4 of the way? 90% of the way?

As we did, in the Prime Spiral figure, let's create a reference number in cell C1, between 0 and 1

What do we need to change?

Everywhere we previously said (A+B)/2, we need to change this. See the end of this document for further reference.

A general formula for the new point is:

(Distance between Two Points) (% Move) + Start Point

or

(End Point — Start Point) × (% Move) + Start Point

or

=(b5-b4)*c1+b4

Copy this to cells b8:c10, and redo the process, as we did above.

IDEA 2: ***Create an Animation to Show these new Distances***

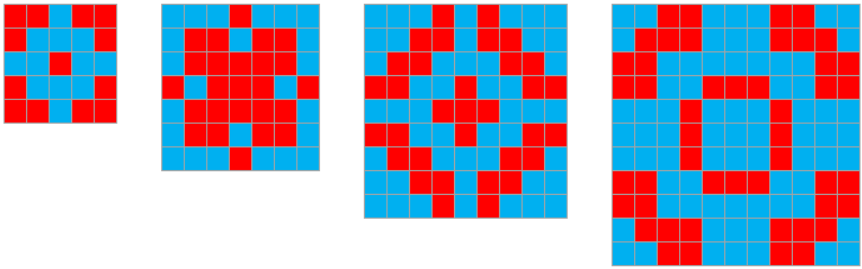
Are we done? Maybe. Create a short macro to go from 0 to 1, showing all these images.

The macro could step from 0 to 100 and then divide each number by 100. Or the macro could step from 0 to 1, but then you must tell it to "Step 0.01"

SYMMETRIC DESIGNS

GOAL

To create a random and symmetric design.



STEP 1: ***Create the Random Part of the Grid***

As you can see, all cell entries are based on only the six numbers in STARTING GRID (far left). ***We're now going to be working in the "Working Grid"***

Let's randomize what we put in these six cells with our familiar

=if(rand()>0.5,1,0)

formula. This returns, with equal probability, 0 or 1.

STEP 2: ***Finishing the Grid***

Use Cell references to make the entire Working Grid symmetric. That is, because our grid is symmetric, each empty cell is now equal to one of these six cells. We merely have to say =(whatever cell) for each of these empty cells.

Follow the outline as shown on the display. For example, because of symmetry, cell M18 is the same as L19. Therefore, in cell M18 belongs the formula:

=L19

But these are all *numbers*, and we want *colors*. We're going to conditionally format our cells to show everything as either black or white.

What we want is to make both the font – and the background – black – IF the cell value is 1. On the other hand, if the cell value is 0, we want both the font and background to be white.

STEP 3b: *Creating Black Cells*

Highlight the entire "Working Grid", and Under the Home menu option is the pull-down menu "Conditional Formatting".

We want to "Highlight Cells Rules", and select "equal to" ***1***

with: (custom format):
make the font color BLACK
fill the cells with BLACK

In doing this, you can still see the zeros. Let's make these zeros WHITE, by doing the same thing as above.

STEP 3c: *Creating White Cells*

We want to "Highlight Cells Rules", and select "equal to" ***0***

with: (custom format):
make the font color WHITE
fill the cells with WHITE

OTHER IDEAS

What else? Here are some ideas to play around with:

IDEA 1: *Making Our Grid Bigger*

Are we done? Of course not. How many different symmetric designs are there here? What happens if we add a column to both sides (and a row to the top and bottom)?

We now have 10 main cell entries.

IDEA 2: *Creating a Binary to Decimal Name for our Graphic*

If we combine these 10 cell entries of 1s and 0s, we have a 10-digit binary number. How could we change the nomenclature to make this a decimal number?

THE PRIME SPIRAL

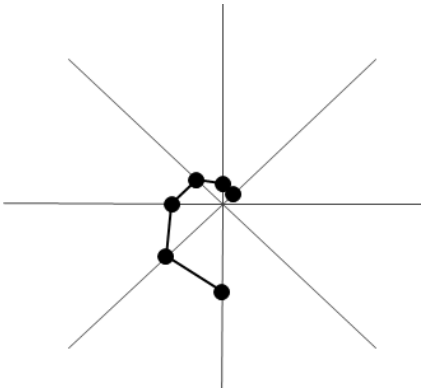
AND MOVING ABOUT THE GRID

I want to "move about" the grid by creating evenly-spaced angles, and then move up an imaginary line on that angle by the distance of the next prime number.

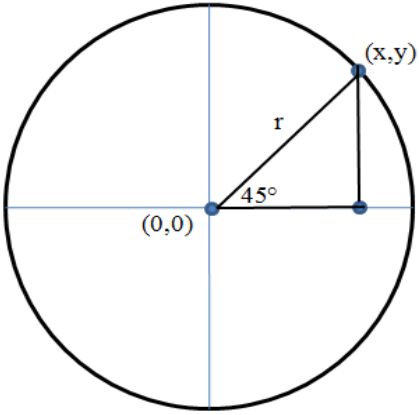
For example, if I evenly spaced out the angles by 45 degrees and started plotting the prime numbers, *what would happen?*

Here's a start:

Prime	Angle
2	45
3	90
5	135
7	180
11	225
13	270



There are really two ways I can visualize these points. One is by the distance and angle (r, α). The other is by moving relative to the x-and y-axis. These two methods of moving – these coordinate systems – are:



Coordinate Systems	
POLAR	CARTESIAN
(r, α)	(x, y)

But how do go from Polar Coordinates (which I know) to Cartesian Coordinates (which I don't)? How do I find (x, y)? A bit of trigonometry helps in this regard, as I know the following basic relationships (using 45° as an example):

$$\sin(\theta) = \frac{opposite}{hypotenuse} \qquad \cos(\theta) = \frac{adjacent}{hypotenuse} \qquad \tan(\theta) = \frac{opposite}{adjacent}$$

It seems simple enough: substitute *x*, *y*, *r*, and 45° and I can arrive at sine and cosine relationships, enabling me to solve for *x* and *y*:

$$\cos(45°) = \frac{x}{r} \qquad \sin(45°) = \frac{y}{r}$$

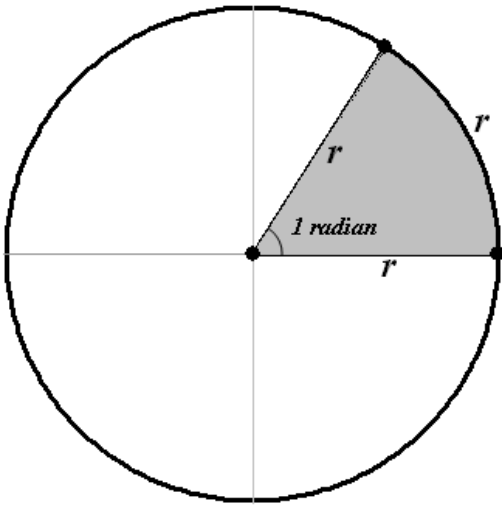
$$\downarrow \qquad\qquad \downarrow$$

$$x = r\cos(45°) \qquad y = r\sin(45°)$$

But Microsoft doesn't use degrees – it uses ***radians***. *What are radians* – and how do I convert degrees *to* radians? Let's find out.

DEGREES TO RADIANS

Up to this point, we've talked about "angles" in terms of "degrees". 45˚. 30˚. 60˚. Many problems talk about the angle being "π/2 radians". What's the meaning of this?

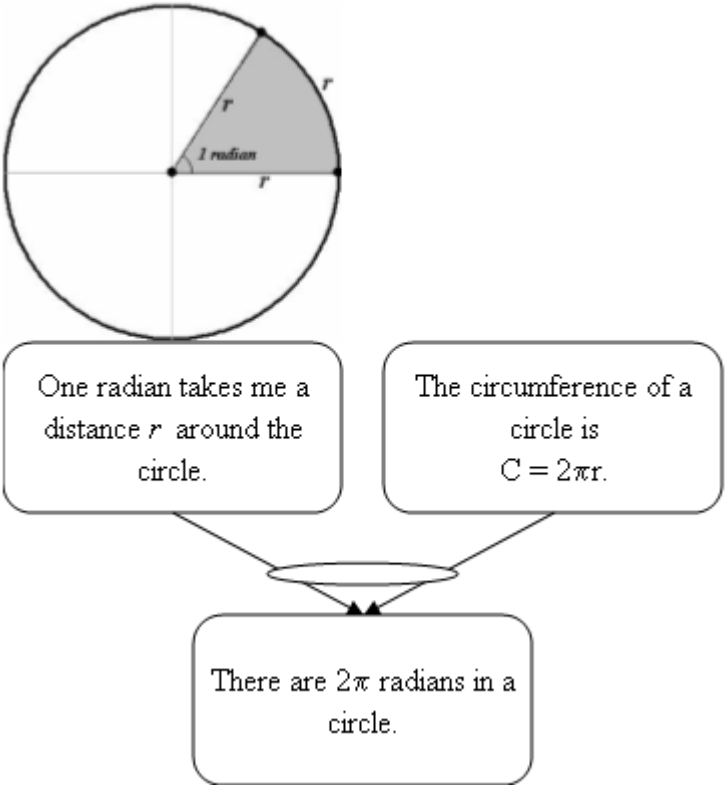


Look at the following circle with radius *r*. I've got three distances labeled *r* here. Two actually *are* the radius, defined by the

distance from the center of the circle to the outside. The other is that same distance, but this time measured as we move *around* the circle.

Now, the question is: how many degrees have I measured when I move the distance *r* around the circle? Answer: ***1 radian.***

"Radian" is a degree measure. It's less than 90˚, but more than 45˚ … you can tell just by looking. But *how* far exactly? Let's see.



So *if* I move 2π radians around the circle, *then* I'm back where I started. 360˚.

π radians and I'm only *halfway* around the circle: 180°

What's the general method of converting between radians and degrees?

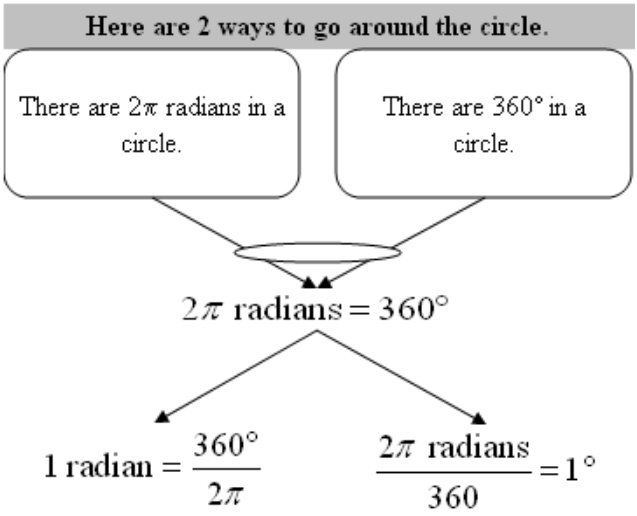
You really don't have to remember any formulas. Instead, think of this: there are two ways of going all the way around the circle:

1. by radians, and we know there are 2π radians in the circle.
2. by degrees, and we know there are 360° in a circle.

Since we're going around the circle, these two must be equal. Therefore:

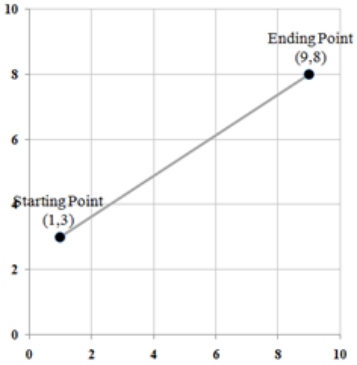
$$\mathbf{2\pi \text{ radians} = 360^\circ}$$

Therefore:



MOVING A CERTAIN DISTANCE BETWEEN POINTS

Suppose I want to move a certain distance between two points. Let's say the starting point is (1,3), the ending point (9,8), and I want to go 75% (3/4th) of the way. *What's my new point?*



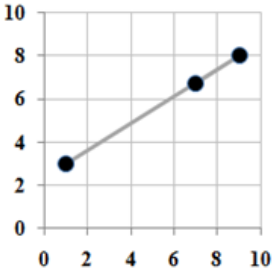
The x-distance between the two points is (9 - 1) = 8 and the y-distance between the two points is (8 - 3) = 5

I want to move 75% of the way, starting at "Starting Point" and moving to "Ending Point".

The x-distance moved is (9 - 1)(0.75)

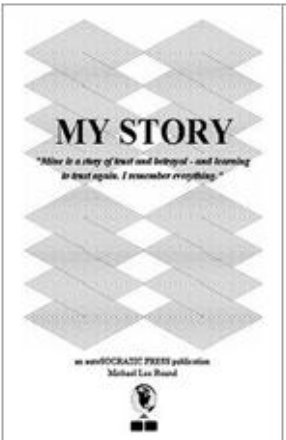
and the y-distance moved is (8 - 3)(0.75)

The "Starting Point" is (1,3).



My new coordinates are:

$x = 1 + (9 - 1)(0.75) = 7.00$
$y = 3 + (8 - 3)(0.75) = 6.75$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"Mine is a story of trust and betrayal - and learning to trust again. I remember everything."
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From Dot to Dot
and everything in-between.
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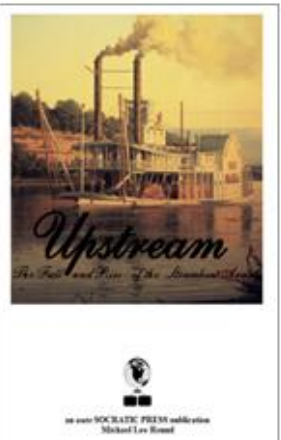
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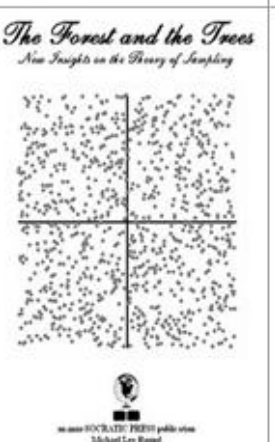
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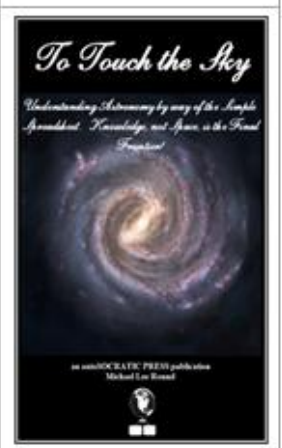
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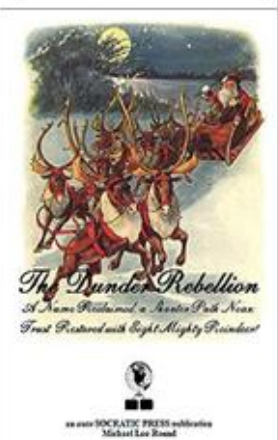
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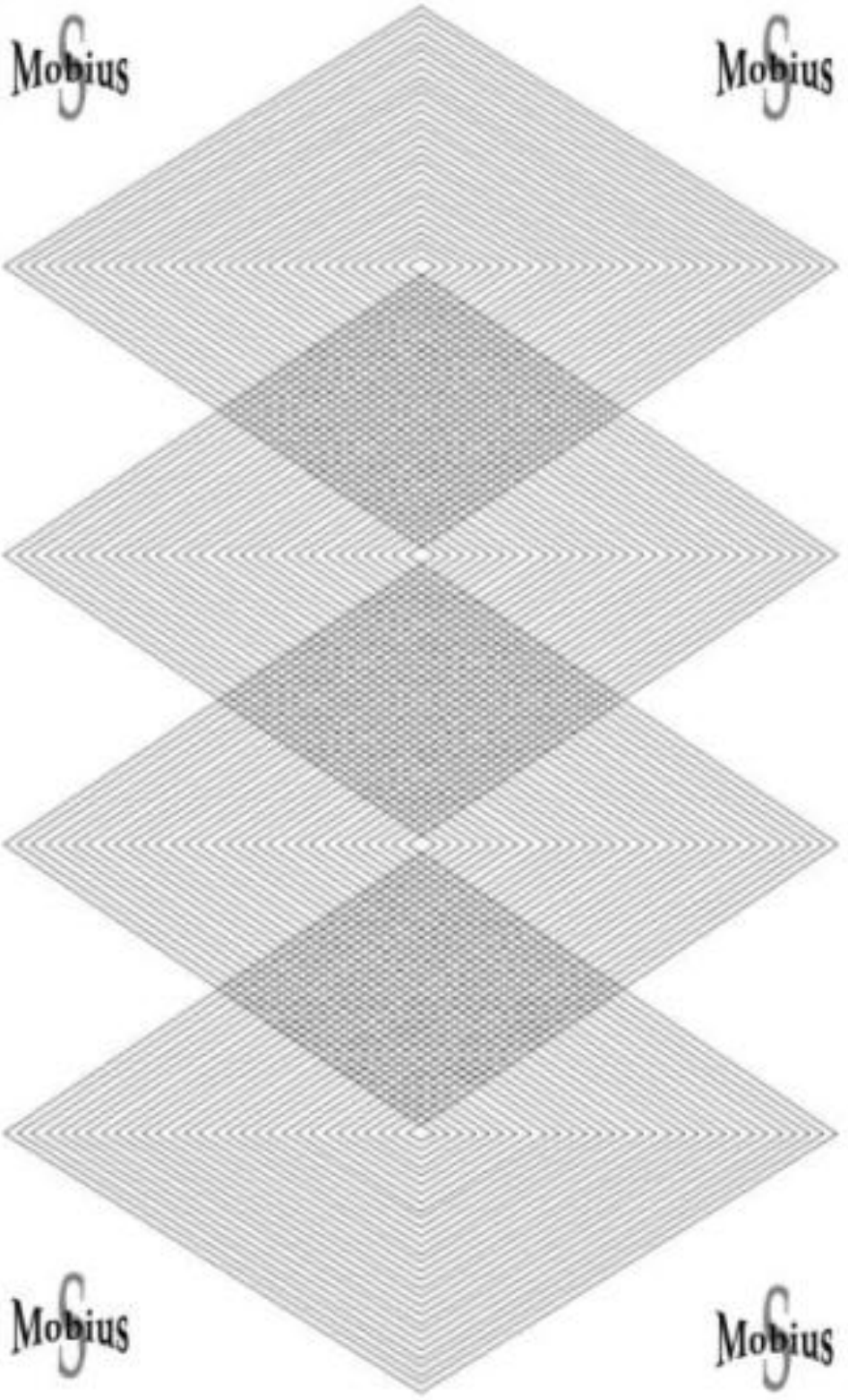
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