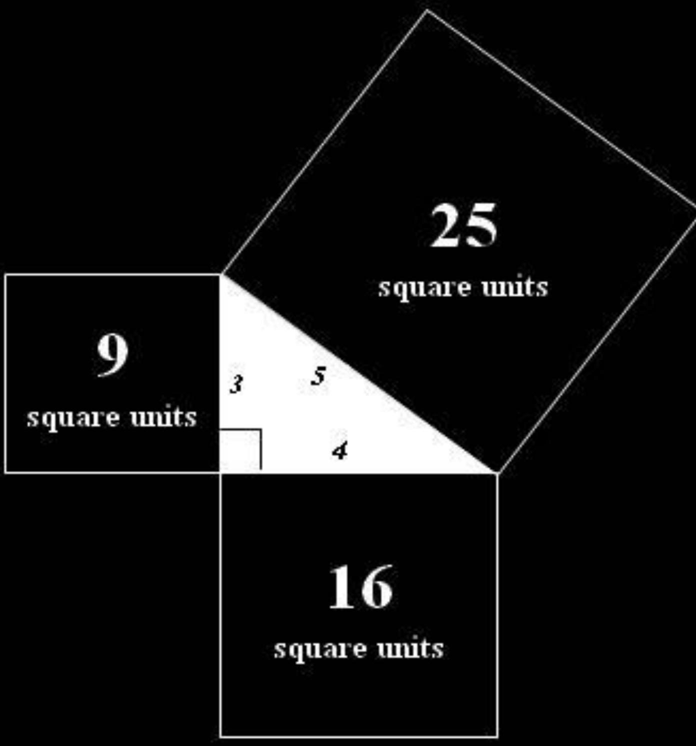
Second Wind

The Myth of Exhaustion: What's Possible When the "Second Wind" Barrier is Breached



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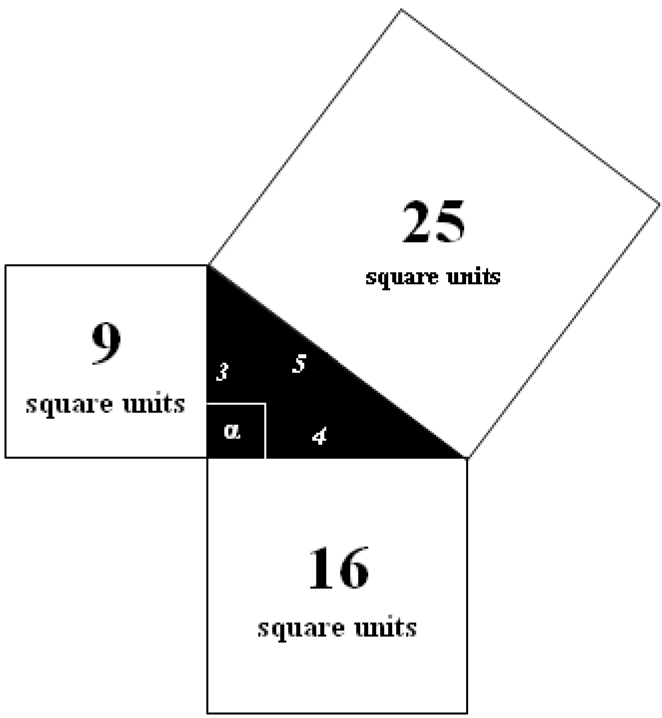
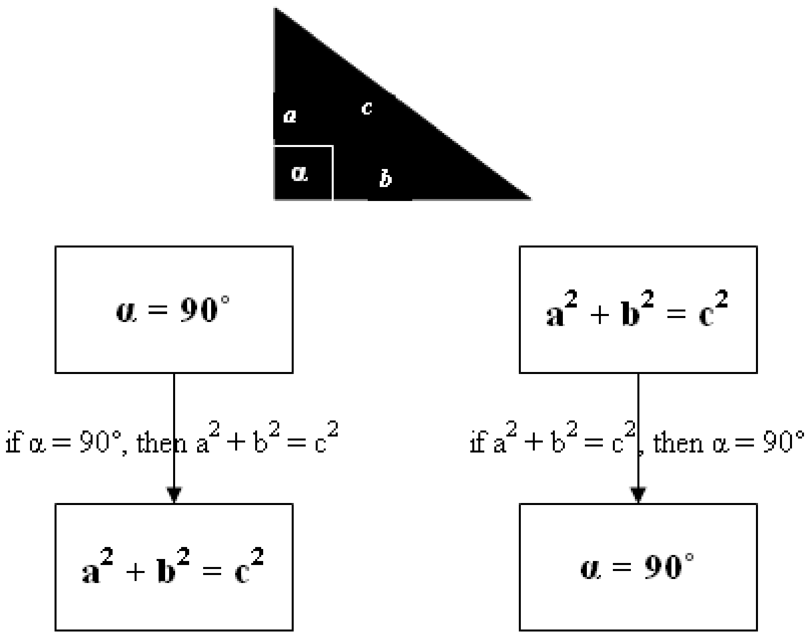
PART I

The Pythagorean Theorem
A Brief Introduction



$$3^2 + 4^2 = 5^2$$

The Pythagorean Theorem not only tells us if the central angle is a right angle (90˚), then the relationship $a^2 + b^2 = c^2$ holds, but the reverse as well. That is, if I have a triangle where $a^2 + b^2 = c^2$, then the central angle is a right angle.

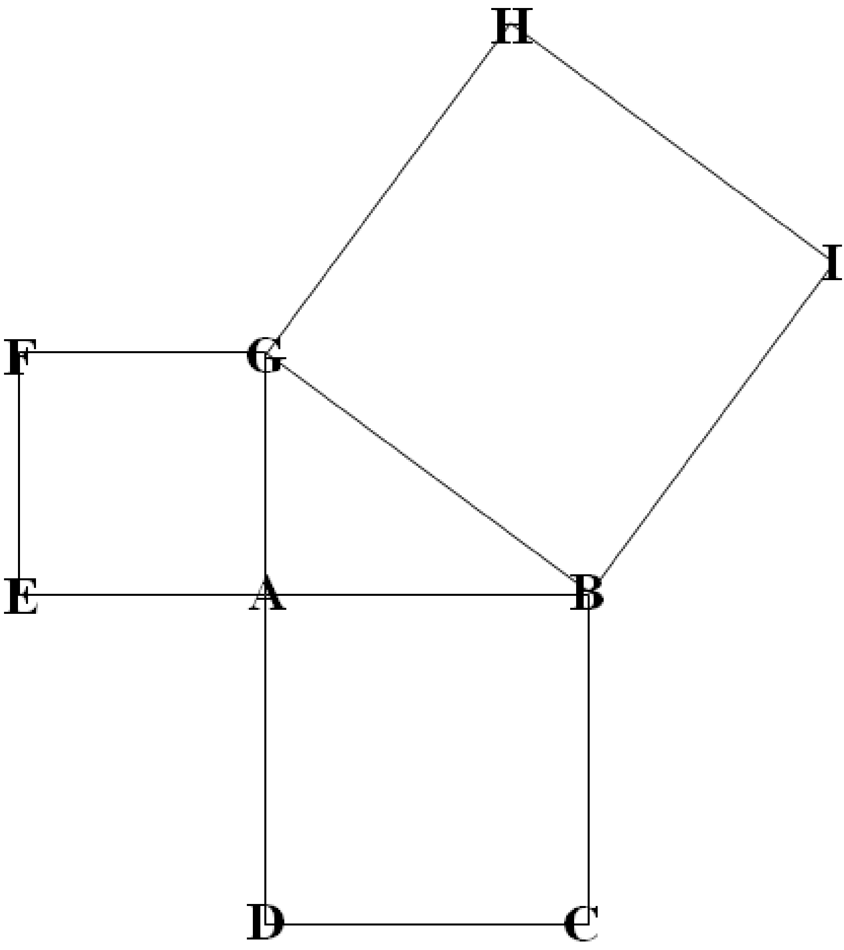


But these simple thoughts led to a simple – and profound – question: what happens to the diagram when the angle is *not* 90˚?

How would I graph all of this?

Our Situation On A Graph

Before I get started, let's label all the points on our Pythagorean Theorem Image with points A – I.



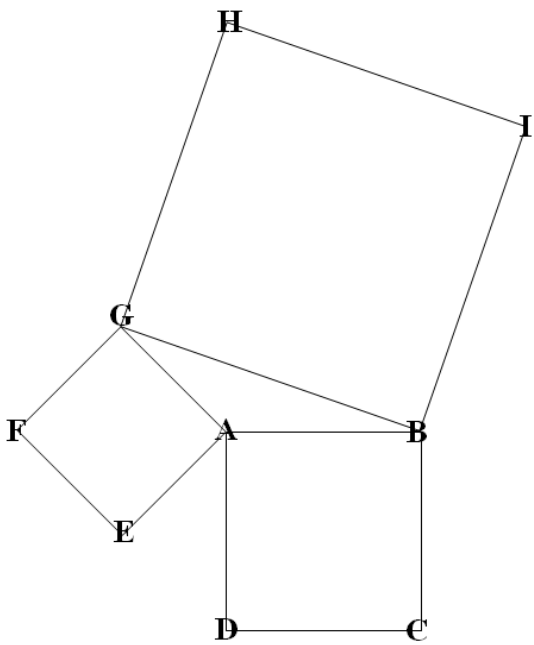
Points A, B, C, and D should be easy to find. This square does not change at all, and I know the length of each side. For simplicity sake, I'll make a length '4' equal 400 on my scatter plot diagram, and center this square with the following coordinates:

Point	x	y
A	500	900
B	900	900
C	900	500
D	500	500

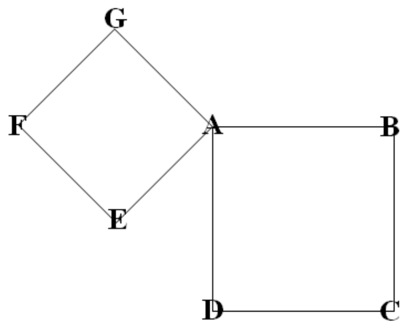
Let's get started on our square with length 3 (now 300). How can I find the coordinates for this square?

First off, let's tip the square to get a realistic look at what we're confronting. After all, that's the question that got this train of thought going in the first place.

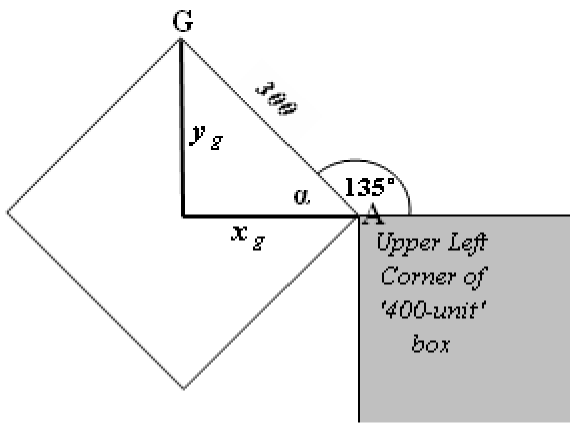
Instead of this square being at 90°, let's tip it to 135°. What does this look like?



POINT G



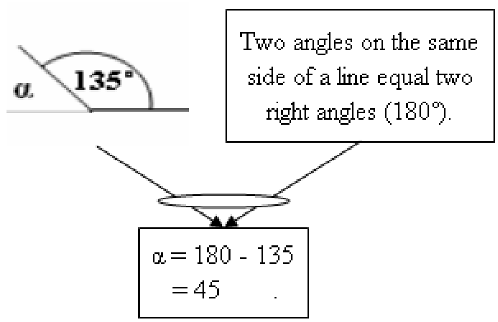
I want to find the coordinates of Point G (x_g, y_g). What do I know about anything? I'm assuming I know the angle I've tilted the 300-unit box, so that's a start. I can extend a line from B past A, and, dropping an altitude line down from G, create a triangle within my 300-unit box:



But do I know anything about the angle within the triangle? Do I know anything about the distances of the segments of the triangle? Just one right now: the hypotenuse has length 300.

What about the other items?

I know if I extend the line from B to A, I will have 180 degrees on one side of the line. This is bisected by one angle of 135 degrees, meaning the remaining angle is 45 degrees.

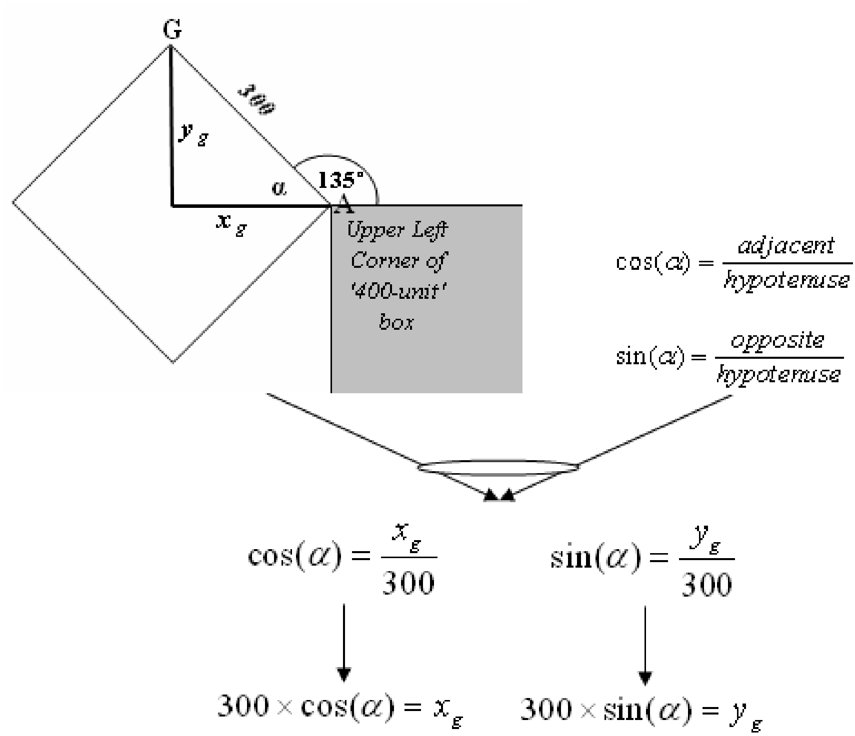


Knowing this, the fact the length of my box of length ‘3’ is really 300 according to my scatterplot, and a bit of trigonometry, I may be on my way.

The two basic trigonometric functions I’m interested in are as follows:

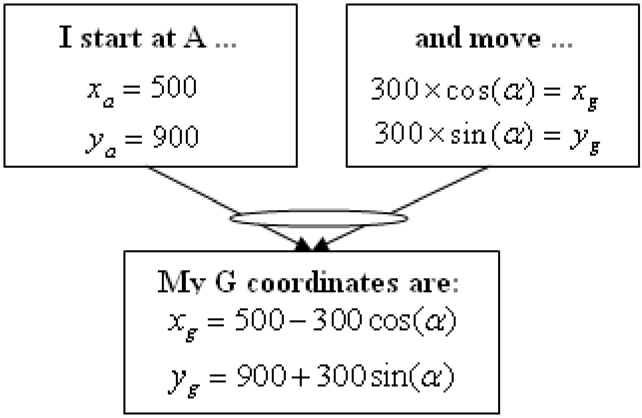
$$\cos(\alpha) = \frac{adjacent}{hypotenuse} \qquad \sin(\alpha) = \frac{opposite}{hypotenuse}$$

Let’s put all of this information together now:

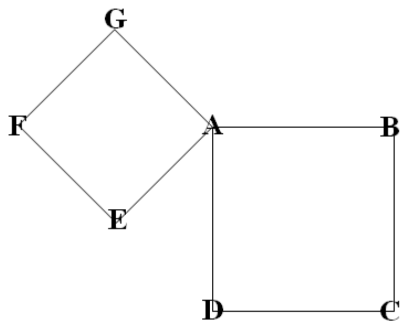


What do I know now? I know x_g and y_g, but what *are* they? Do they allow me to plot point G? They help, yes. However, they are not points. They are merely distances. I must add these distances to a point I already know to be able to plot Point G.

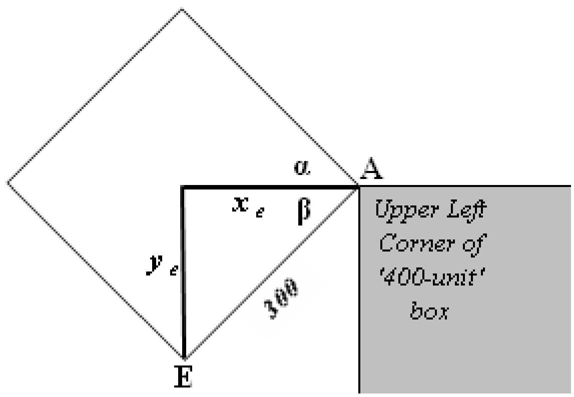
I know the coordinates of Point A. Let’s use it:


I start at A ...
$x_a = 500$
$y_a = 900$
and move ...
$300 \times \cos(\alpha) = x_g$
$300 \times \sin(\alpha) = y_g$
My G coordinates are:
$x_g = 500 - 300\cos(\alpha)$
$y_g = 900 + 300\sin(\alpha)$

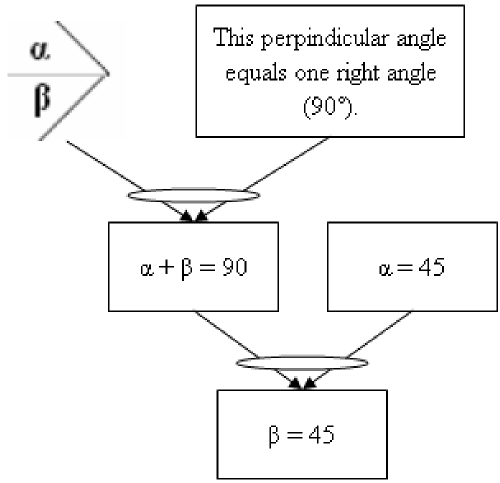
POINT E



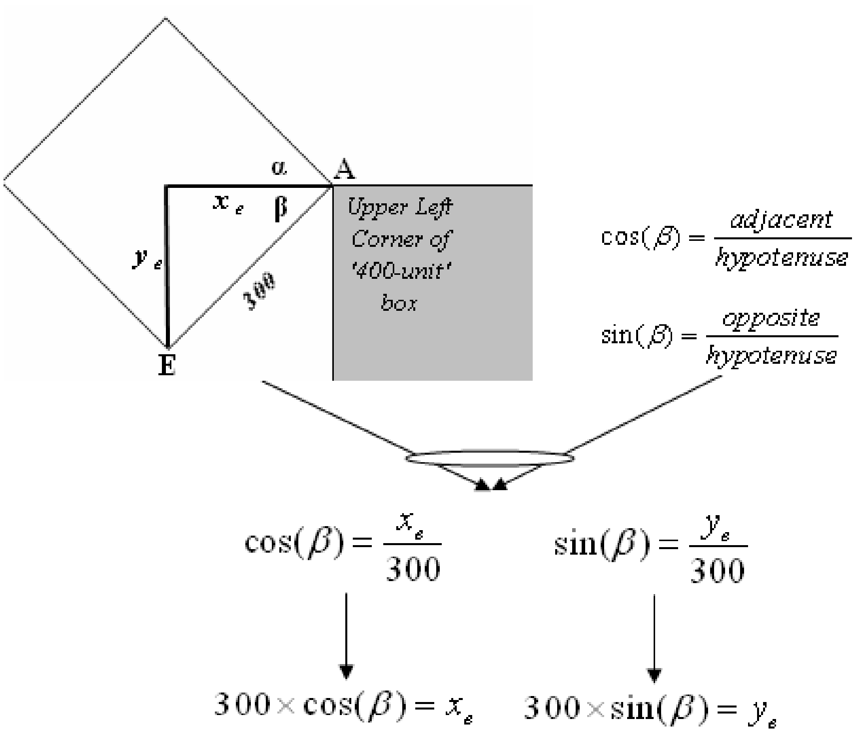
I next want to find the coordinates of Point E (x_e, y_e). What do I know about anything? I've already determined angle α above, and I know angles α + β together form a right angle. Therefore, I can find β. Knowing this and the same trigonometry from above, likely I can find the distances needed to plot the coordinates for Point E. Let's see.



Can I first find the angle β?

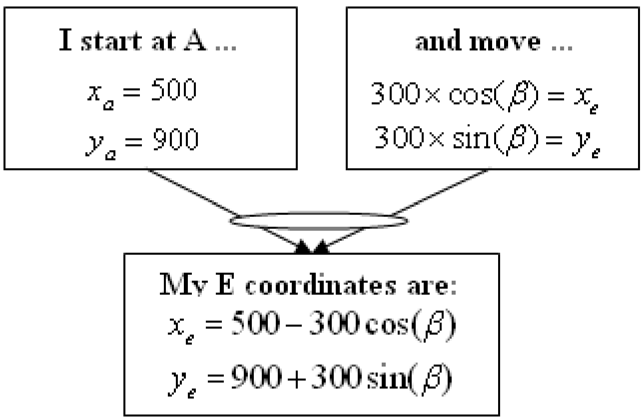


What about the lengths of the sides of the triangle?



What do I know now? I know x_e and y_e, but what are they? Do they allow me to plot point E? They help, yes. However, they are not points. They are merely distances. I must add these distances to a point I already know to be able to plot Point E.

I know the coordinates of Point A. Let's use it:



A Time-Out To Graph

Before going any further, let's see if our results make sense. We've got 6 points (A, B, C, D) on the 400-unit square, and 2 points (G and E) on the 300-unit square.

Let's graph the results and see if we're on the right track.

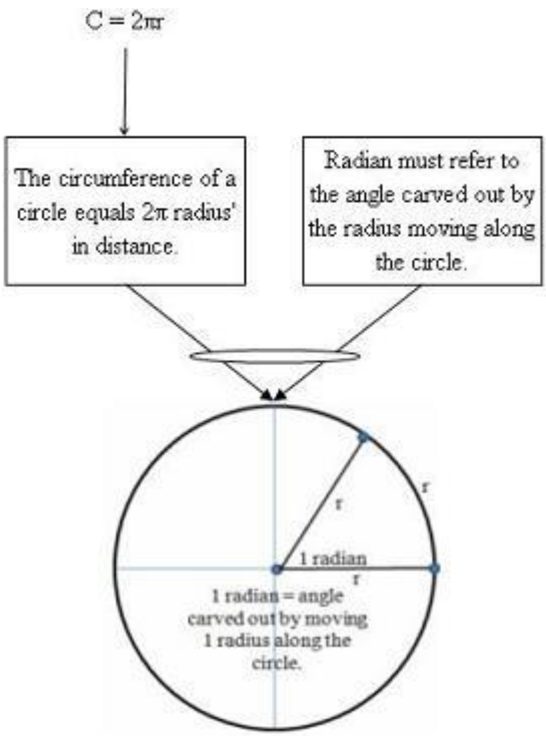
But right away, I see something is amiss. Everything I've done thus far depends on the initial angle I'm bending the 300-unit box. This has been measured in degrees. Excel does not use 'degrees'; it uses 'radians'.

Let's make this change – but how?

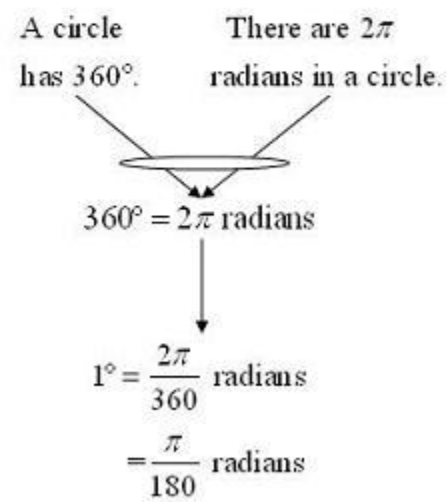
Degrees to Radians

I know something about the circumference of a circle, and I know this formula includes the circle radius *r*.

But does this lead me anywhere? I'm still talking about "distance", while I'm looking for something regarding "angle" or "degree".



Radian, then, must refer to the angle carved out by the radius along the perimeter of the circle. And if one radian carves out one radius, and there are 2π radii on the circumference, then there are 2π radians in a circle.

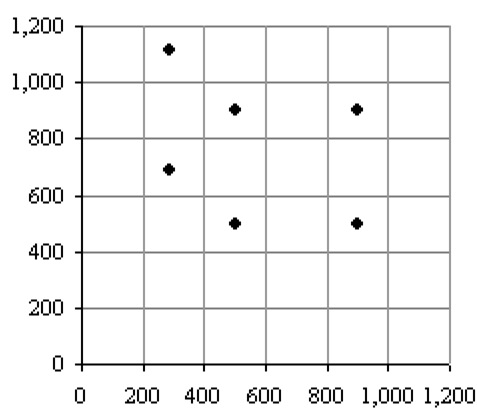


I'm closing in on the answer to my question*: how do I translate degrees into radians?* Above, I gave an expression for one degree, but I don't have one degree. I have lots of different degrees. Fortunately, the translation is now easy.

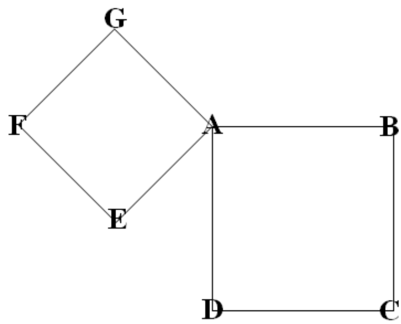
RELEVANT DEGREES

tilt	**alpha**	**beta**
135	45	45

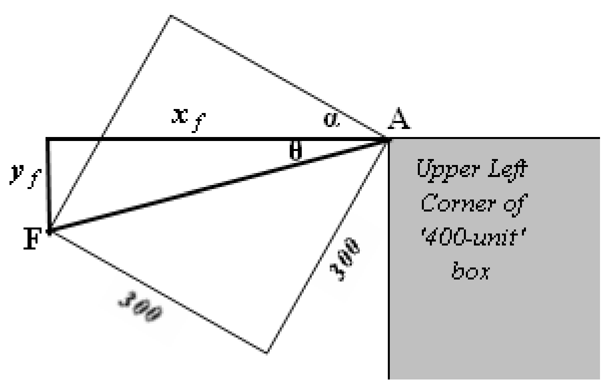
Point	x	y
A	500	900
B	900	900
C	900	500
D	500	500
G	288	688
E	288	1,112



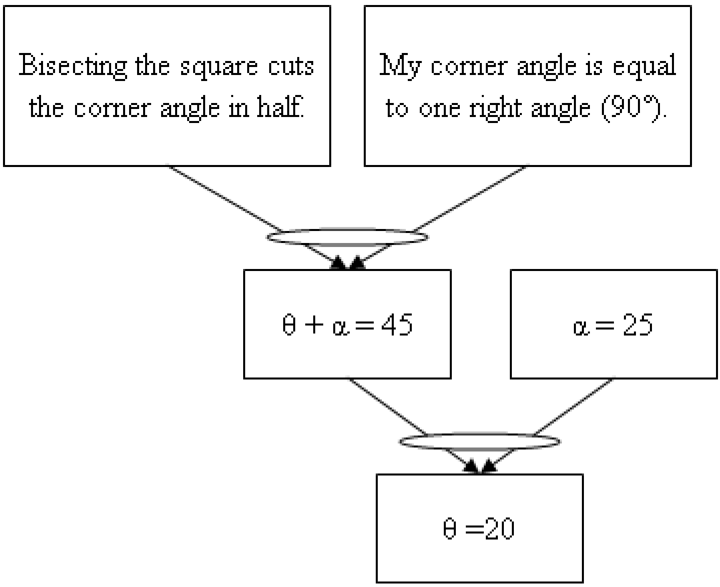
POINT F



I want to find the coordinates of Point F (x_f, y_f). What do I know? Well, first my graphic above is misleading, because it seems like it's on a direct line with a line passing through AB. That's not necessarily the case, so let's modify the graphic a bit.



If I draw a line directly from Point A to Point F, I've bisected that perpindicular angle, and I know one of the angles already in that corner. This gives me the angle to a triangle created by reaching Point F with only straight lines. Remembering I've now changed my tilt to 155 degrees to show the difference between Points F and A, and with a bit of logic, I can arrive at my angle θ:



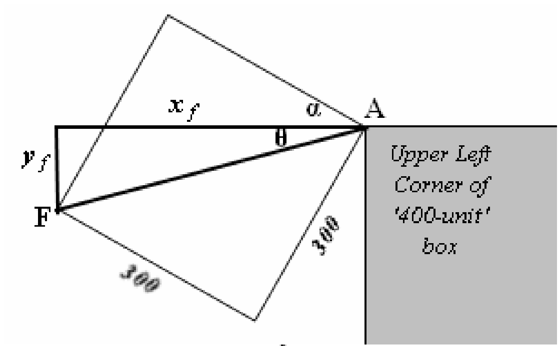
and my hypotenuse …

$$300^2 + 300^2 = x^2$$

$$\downarrow$$

$$x = 300\sqrt{2}$$

And carrying out the now obvious logical steps, I can find the coordinates of Point F:



$$\cos(\alpha) = \frac{adjacent}{hypotenuse}$$

$$\sin(\alpha) = \frac{opposite}{hypotenuse}$$

$$\cos(\theta) = \frac{x_f}{300\sqrt{2}} \qquad \sin(\theta) = \frac{y_f}{300\sqrt{2}}$$

$$300\sqrt{2} \times \cos(\theta) = x_f \qquad 300\sqrt{2} \times \sin(\theta) = y_f$$

And, applying the same logic as before, I arrive at the coordinates for Point F:

I start at A ...

$$x_a = 500$$

$$y_a = 900$$

and move ...

$$300\sqrt{2} \times \cos(\theta) = x_f$$

$$300\sqrt{2} \times \sin(\theta) = y_f$$

My F coordinates are:

$$x_f = 500 - 300\sqrt{2}\cos(\theta)$$

$$y_f = 900 + 300\sqrt{2}\sin(\theta)$$

Putting it All Together

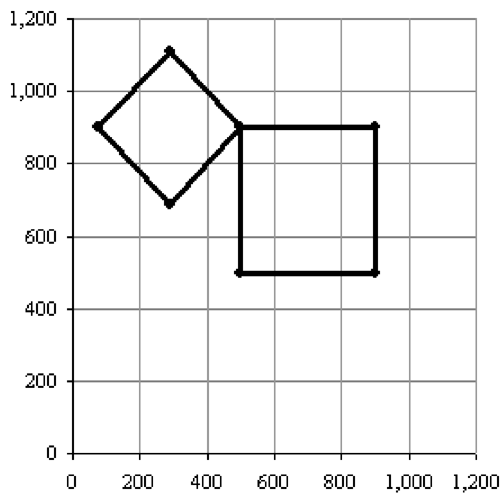
Applying these new coordinate formulas to Point F should round out two squares. Let's see.

But to make the squares properly connected, I need to do more than plot the points and connect the dots. I need to have the lines come back to a meeting point to move on to the second square. That's easily done:

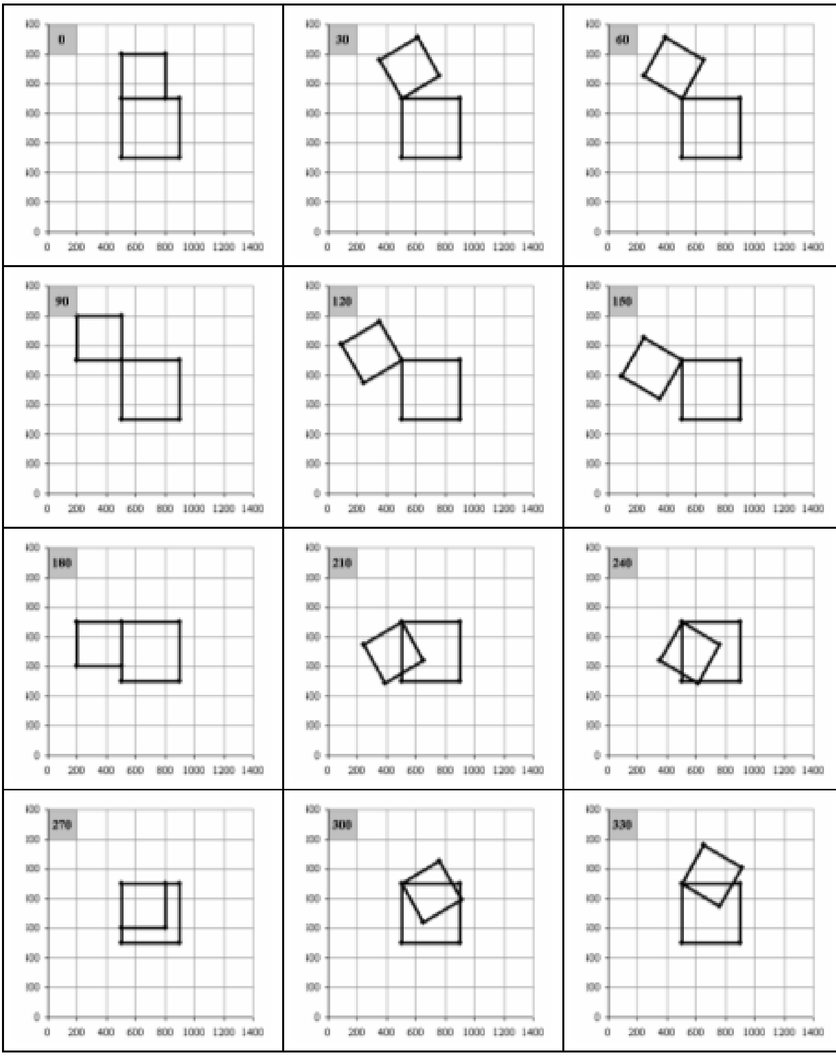
RELEVANT DEGREES

tilt	135
alpha (α)	45
beta (β)	45
theta (θ)	0

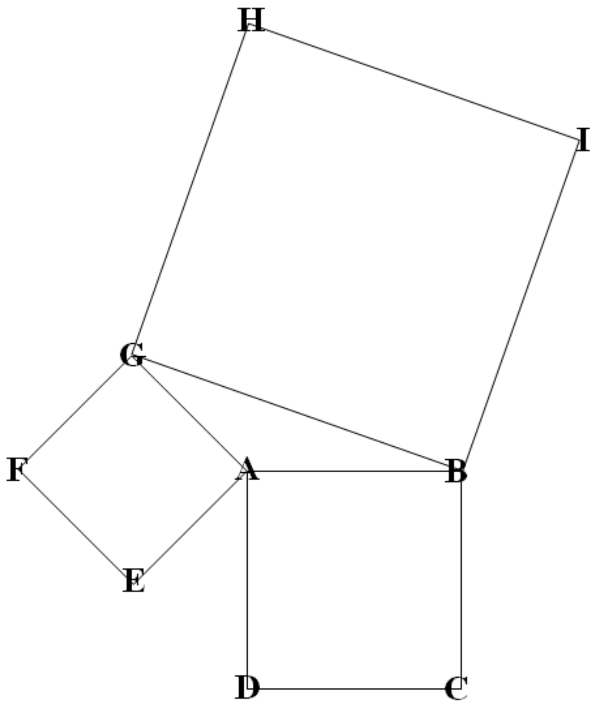
Point	x	y
A	500	900
B	900	900
C	900	500
D	500	500
A	500	900
G	288	1,112
F	76	900
E	288	688
A	500	900



Tilting Our "300-Unit" Square
0 - 360 Degrees

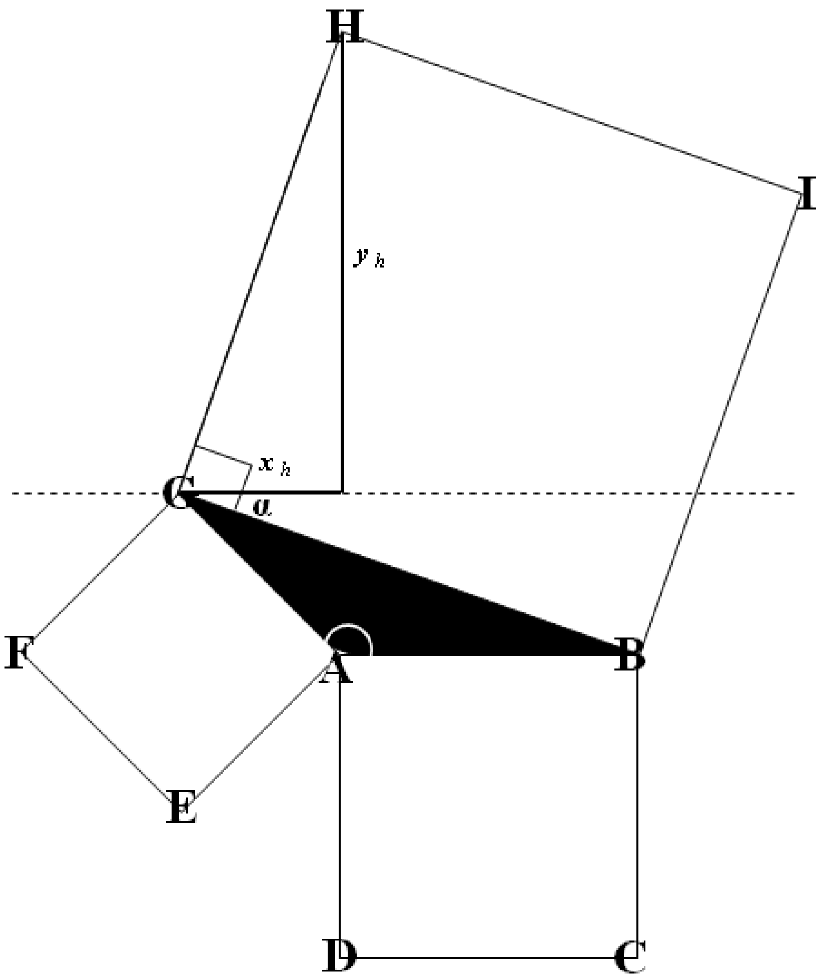


POINT H



I'm almost home – two points left: H and I. Like points G, F, and E, I hope it's as easy as extending a line, dropping an altitude line to a point, using some easy geometry theorems, and applying basic trigonometric formulas. We'll see.

But I see a problem immediately. Where do I extend a line from in moving towards the coordinates of Point H?



Let’s try some things. How about extending a line through G, but parallel to AB. Drop an altitude line from H, and I’ve got my triangle. But do I know anything about the angle involved?

I know angle HGB is a right angle, because this is a square shape. But I’m not interested in the whole angle, just a part of it.

Do I know anything about the angle α under the new extended horizontal line? I don't see what.

What about the angle I do know something about in all this – the original tilt angle located at BAG? Using everything I know about opposite and interior angles, I don't see how this helps.

I seem stuck.

I *am* stuck.

And exhausted. I've tried everything, but this one elusive question has me stumped. I can't continue my programming. My search is over.

For now …

PART II

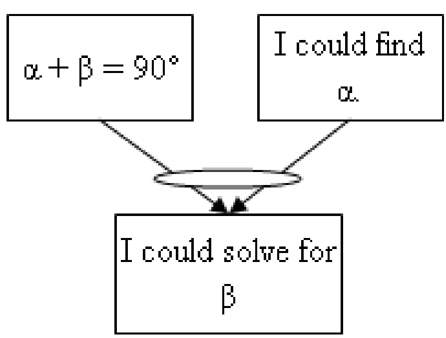
Beyond the Breaking Point
A Mental "Second Wind"

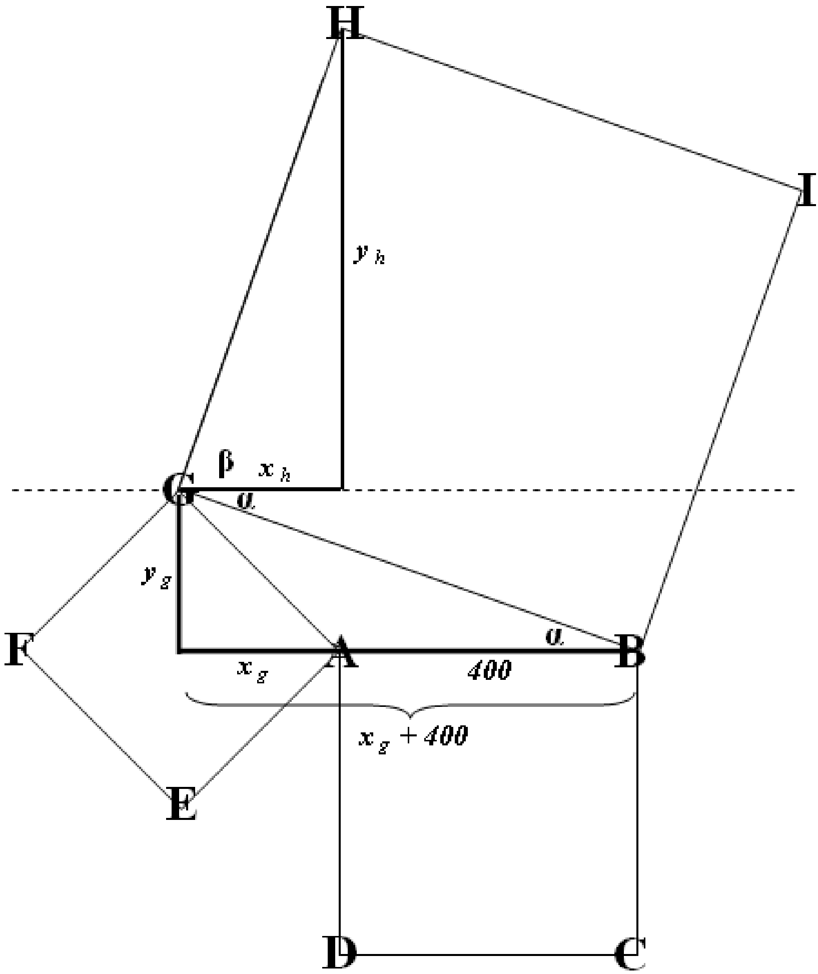
Where does a new idea come from? How does one "think outside the box"?

I don't know.

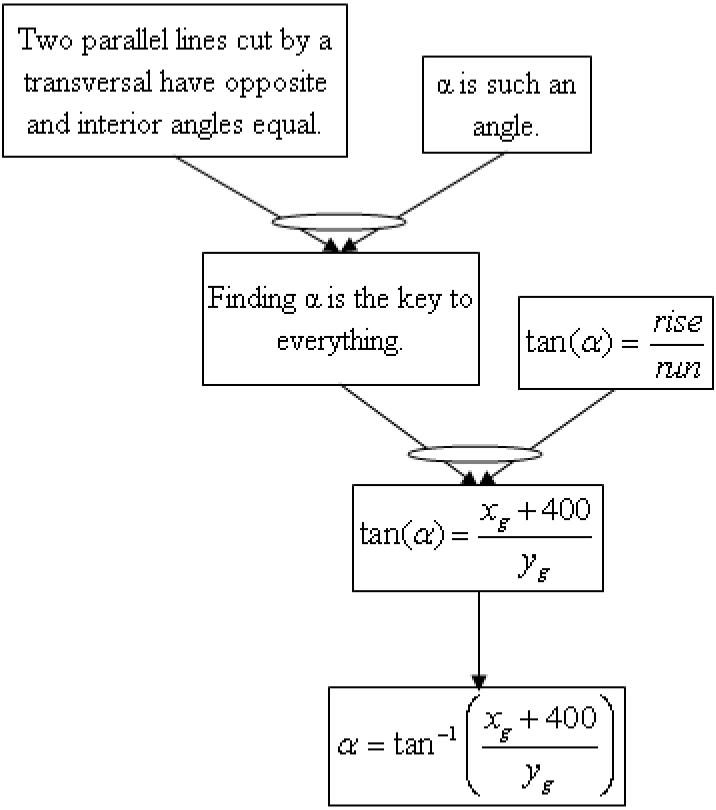
However, after struggling for hours to figure out how to find an angle in my triangle to help me find the H-coordinates, I finally hit upon something.

I need β. I know something about α and β, because this is a right-angle. Therefore, if I could find α, I'm home free. But how? I've been applying the idea of a transversal cutting two parallel lines, but that doesn't seem to be getting me anywhere.

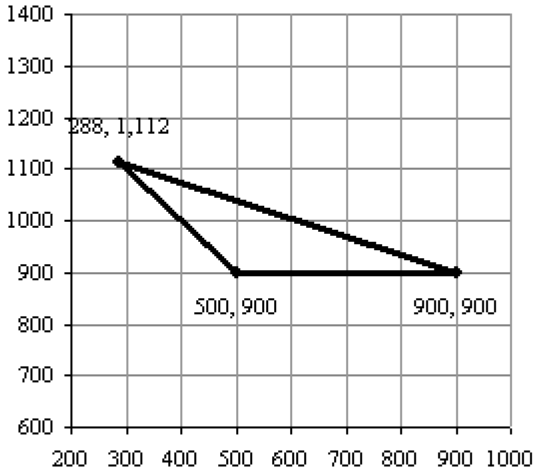




After banging my head against the wall for hours, it finally came to me: I can't get α directly by adding angles together like I have earlier (for example, two angles = 180, or a right angle = 90). However, I do know here the ratio of the length of the sides of my triangle is the same of the tangent of α! The arctangent!



Let's use a concrete example to see how this works: if the tilt is 135˚, above we found the coordinates of G to be (288,1112). Plugging these into the above formula, we have:



$$\alpha = \tan^{-1}\left(\frac{900-288}{1112-900}\right) = \tan^{-1}(0.3464)$$

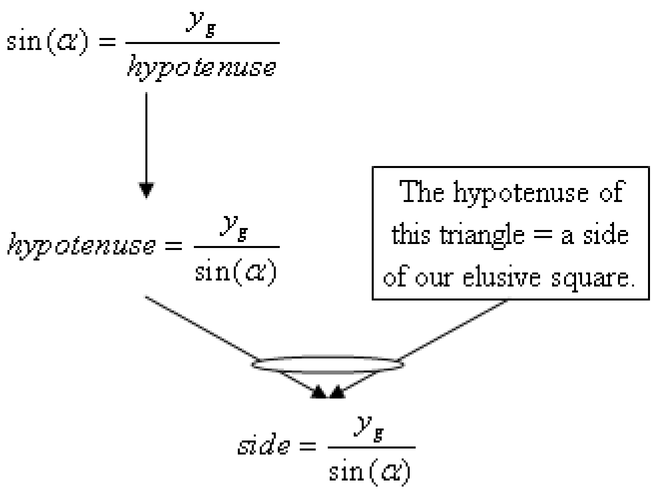
$$\alpha = 0.3335 \text{ radians} \qquad 1 \text{ radian} = \frac{360^\circ}{2\pi}$$

$$\alpha = 19.11^\circ$$

This means I now know what β is (90 – α) = 70.89˚.

Am I home free yet?

Almost. But all I know is the angle. I don't know anything else about my triangle, do I?



As you can tell, things are really starting to get out of control, there are so many formulas in place. Let's get some things formalized in a table to make sure we know what we doing, and to document some of the progress.

Putting it All Together

OK – let's see if we can put all of this together, rather than applying formulas here and there, and see if our graph corresponds to our formulas. In a word, have we done things right?

COORDINATES

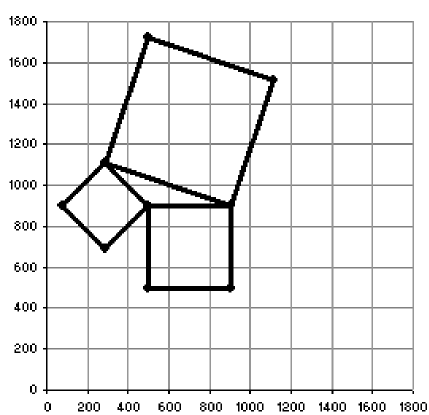
Point	x	y	
A	500	900	*main points, and returning to A*
B	900	900	
C	900	500	
D	500	500	
A	500	900	
G	288	1,112	*E, F, G, and back to A, G*
F	76	900	
E	288	688	
A	500	900	
G	288	1,112	
H	500	1,724	*H, I and back to B, G*
I	1,112	1,512	
B	900	900	
G	288	1,112	

RELEVANT DEGREES for points E, F, and G

tilt	135
alpha (α)	45
beta (β)	45
theta (θ)	0

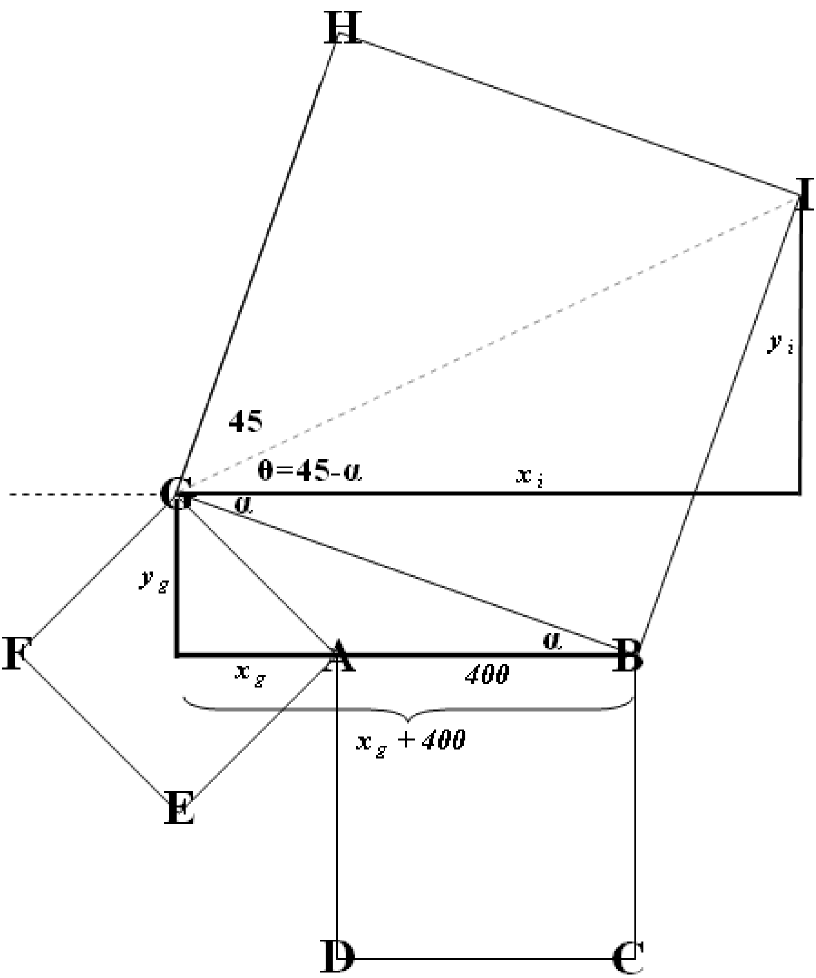
RELEVANT DEGREES for points H and I

rise		212
run		612
pitch		0.347
arctan(rad)		0.33359
arctan(deg)		19.1136
Compl(deg)		70.8864
Comp(rad)		1.2372
length of side		647.847
	x	**y**
H (off G)	212	612
Angle (deg)		25.8864
Angle (rad)		0.4518
	x	**y**
I (off G)	824	400

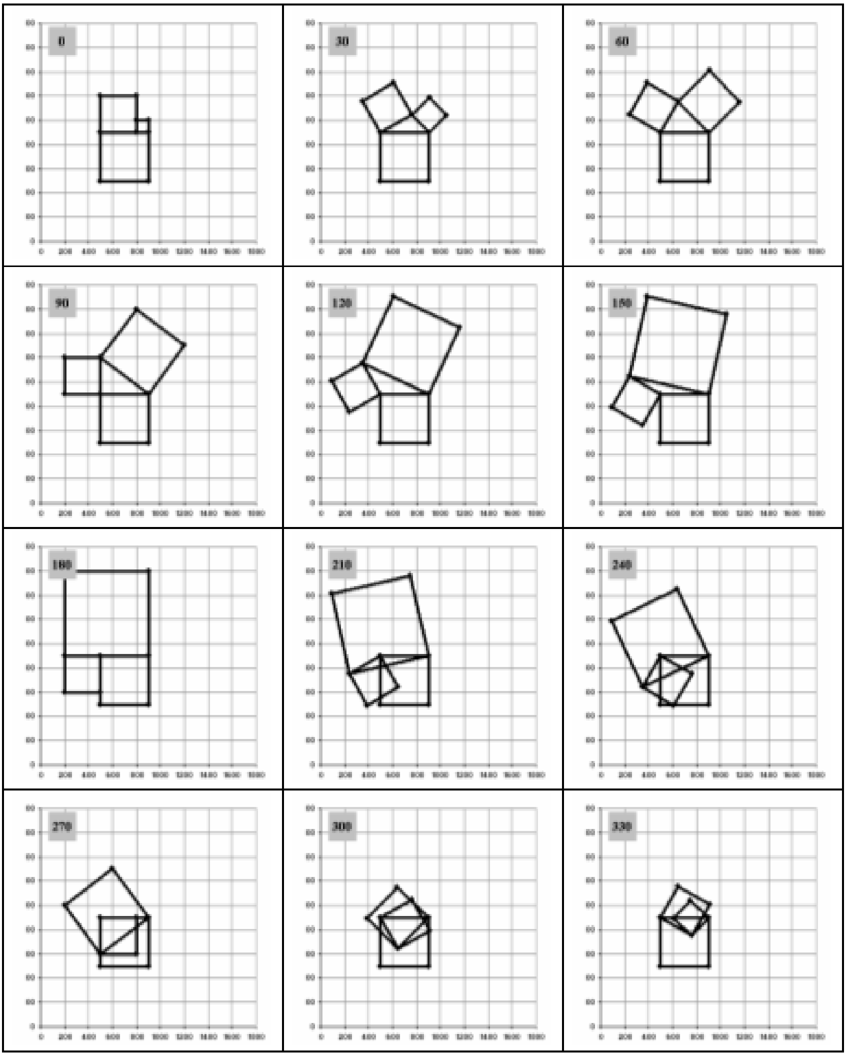


POINT I

I leave it to the reader to complete Point I coordinates. Here is the image I used to complete the table above.

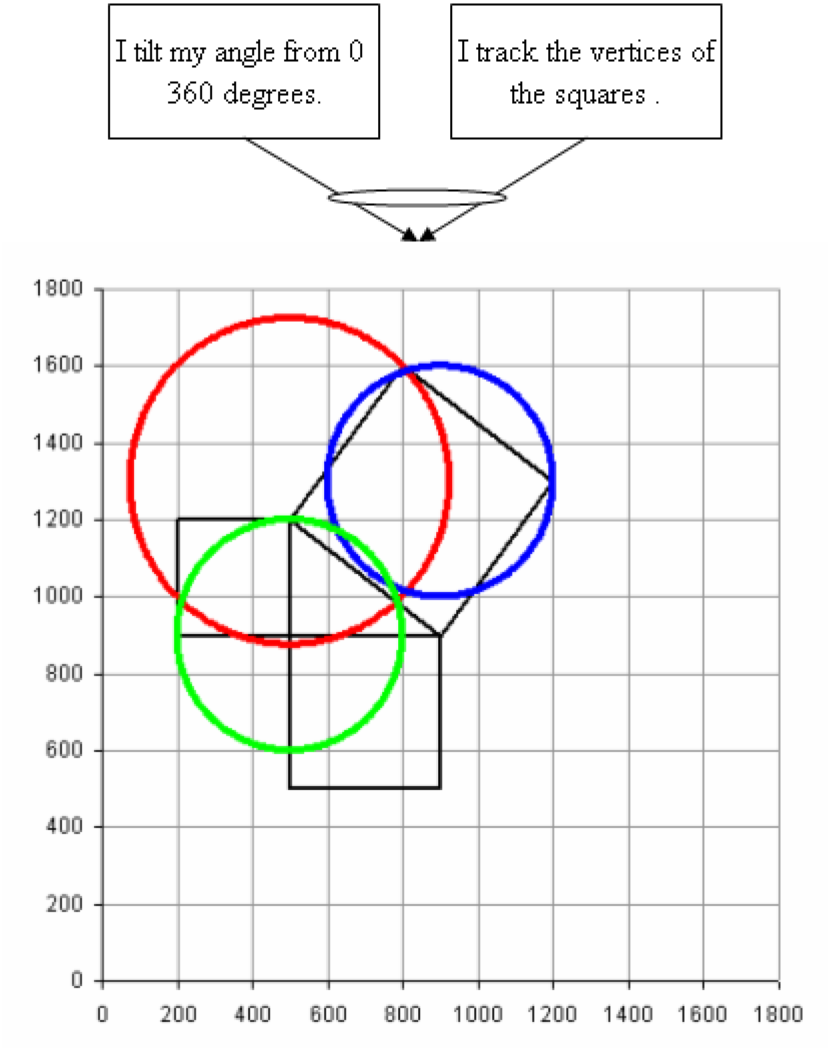


Tilting Our "300-Unit" Square
0 - 360 Degrees



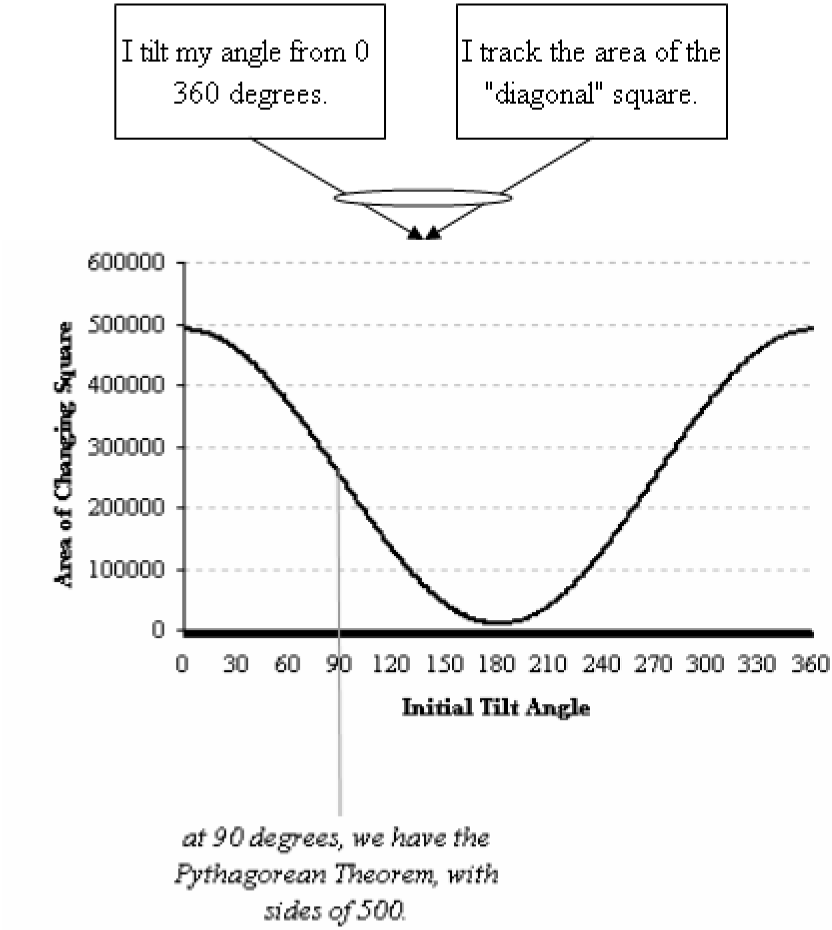
Plotting VERTICES

Tilting the Square from 0 to 360 Degrees



Plotting AREA

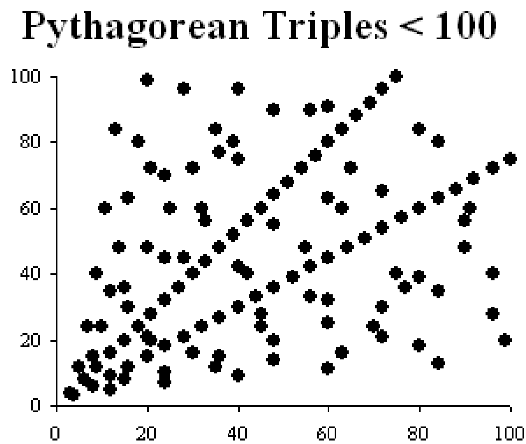
Tilting the Square from 0 to 360 Degrees

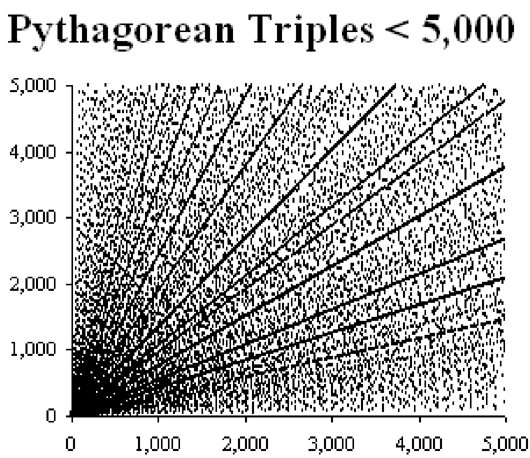


Pythagorean TRIPLES

Plotting Integer Solutions of the Pythagorean Theorem

3	4	5	30	72	78
5	12	13	32	60	68
6	8	10	33	44	55
7	24	25	33	56	65
8	15	17	35	84	91
9	12	15	36	48	60
9	40	41	36	77	85
10	24	26	39	52	65
11	60	61	39	80	89
12	16	20	40	42	58
12	35	37	40	75	85
13	84	85	40	96	104
14	48	50	42	56	70
15	20	25	45	60	75
15	36	39	48	55	73
16	30	34	48	64	80
16	63	65	48	90	102
18	24	30	51	68	85
18	80	82	54	72	90
20	21	29	56	90	106
20	48	52	57	76	95
20	99	101	60	63	87
21	28	35	60	80	100
21	72	75	60	91	109
24	32	40	63	84	105
24	45	51	65	72	97
24	70	74	66	88	110
25	60	65	69	92	115
27	36	45	72	96	120
28	45	53	75	100	125
28	96	100	80	84	116
30	40	50			



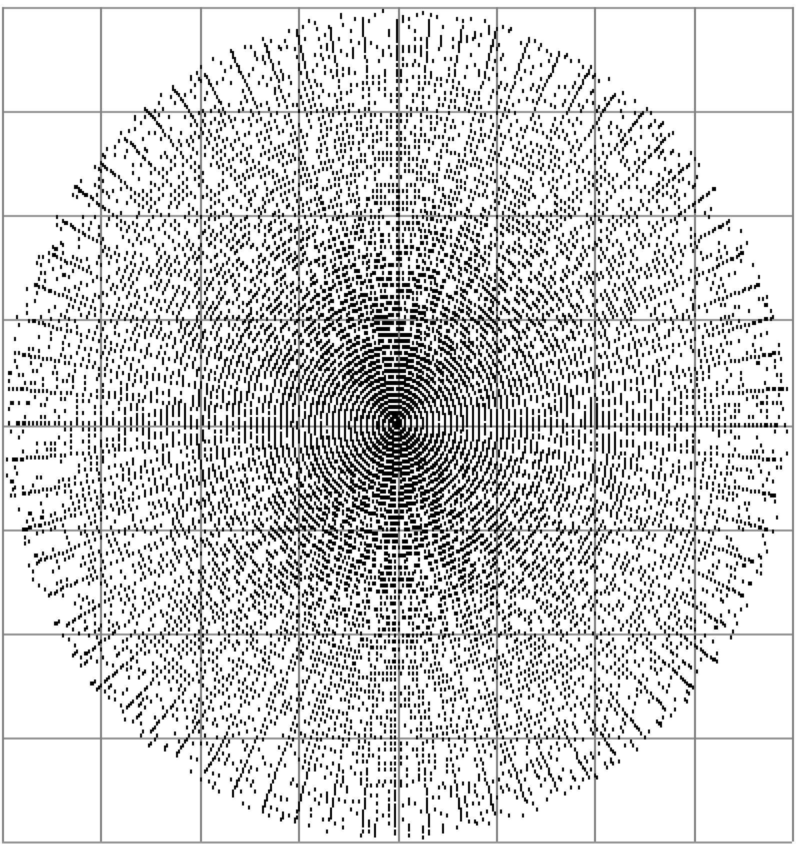


Pythagorean BULLS-EYE

Plotting Diagonals in Polar Coordinates

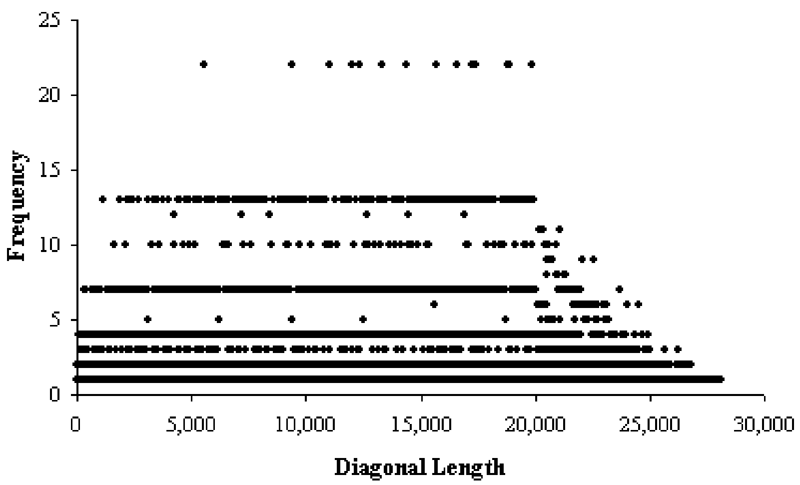
For example: the first Pythagorean triple is 3,4,5. Let’s capture the ‘5’, and give it ordinal 1. The second is 10 (6,8,10). It gets ordinal 2. Etc.

Triple= Angle	Length	Triple= Angle	Length
5	1	20	6
10	2	25	7
13	3	26	8
15	4	29	9
17	5	30	10



Pythagorean FREQUENCIES

Plotting Pythagorean-Diagonals



22 Times	13 Times							
5,525	1,105	5,945	8,245	10,660	13,130	15,145	16,835	18,870
9,425	1,885	6,205	8,585	10,730	13,195	15,170	16,965	18,915
11,050	2,210	6,290	8,840	10,865	13,260	15,370	17,170	18,980
12,025	2,405	6,305	8,845	11,245	13,345	15,385	17,255	19,045
12,325	2,465	6,409	8,905	11,285	13,481	15,457	17,355	19,210
13,325	2,665	6,565	9,010	11,310	13,505	15,470	17,485	19,227
14,365	3,145	6,630	9,061	11,570	13,515	15,555	17,680	19,240
15,725	3,315	6,890	9,265	11,645	13,780	15,665	17,690	19,345
16,575	3,445	6,970	9,435	11,713	13,940	15,805	17,810	19,370
17,225	3,485	7,085	9,490	11,765	13,949	15,860	17,835	19,465
17,425	3,770	7,215	9,605	11,890	14,065	15,990	17,945	19,610
18,785	3,965	7,345	9,620	11,895	14,170	16,095	18,005	19,669
18,850	4,420	7,395	9,685	12,155	14,235	16,133	18,020	19,695
19,825	4,505	7,540	9,805	12,410	14,430	16,165	18,122	19,720
	4,745	7,565	9,860	12,505	14,645	16,250	18,125	19,721
	4,810	7,585	9,945	12,545	14,690	16,354	18,241	19,805
	4,930	7,685	10,205	12,580	14,705	16,385	18,245	19,865
	5,185	7,735	10,335	12,610	14,790	16,405	18,265	19,885
	5,330	7,930	10,370	12,665	14,885	16,465	18,530	19,890
	5,365	7,995	10,455	12,805	14,965	16,490	18,615	19,981
	5,655	8,125	10,585	12,818	15,080	16,705	18,655	
	5,785	8,177	10,625	12,905	15,130	16,745	18,685	

DIAGONAL 5525

has 22 unique $x^2 + y^2 = 5525^2$ combinations

l	w	d	peri	area
235	5,520	5,525	11,280	648,600
525	5,500	5,525	11,550	1,443,750
612	5,491	5,525	11,628	1,680,246
845	5,460	5,525	11,830	2,306,850
1,036	5,427	5,525	11,988	2,811,186
1,131	5,408	5,525	12,064	3,058,224
1,320	5,365	5,525	12,210	3,540,900
1,360	5,355	5,525	12,240	3,641,400
1,547	5,304	5,525	12,376	4,102,644
2,044	5,133	5,525	12,702	5,245,926
2,125	5,100	5,525	12,750	5,418,750
2,163	5,084	5,525	12,772	5,498,346
2,340	5,005	5,525	12,870	5,855,850
2,600	4,875	5,525	13,000	6,337,500
2,805	4,760	5,525	13,090	6,675,900
2,880	4,715	5,525	13,120	6,789,600
3,124	4,557	5,525	13,206	7,118,034
3,315	4,420	5,525	13,260	7,326,150
3,468	4,301	5,525	13,294	7,457,934
3,500	4,275	5,525	13,300	7,481,250
3,720	4,085	5,525	13,330	7,598,100
3,861	3,952	5,525	13,338	7,629,336
4,085	3,720	5,525	13,330	7,598,100
4,275	3,500	5,525	13,300	7,481,250
4,301	3,468	5,525	13,294	7,457,934
4,420	3,315	5,525	13,260	7,326,150
4,557	3,124	5,525	13,206	7,118,034
4,715	2,880	5,525	13,120	6,789,600
4,760	2,805	5,525	13,090	6,675,900
4,875	2,600	5,525	13,000	6,337,500
5,005	2,340	5,525	12,870	5,855,850
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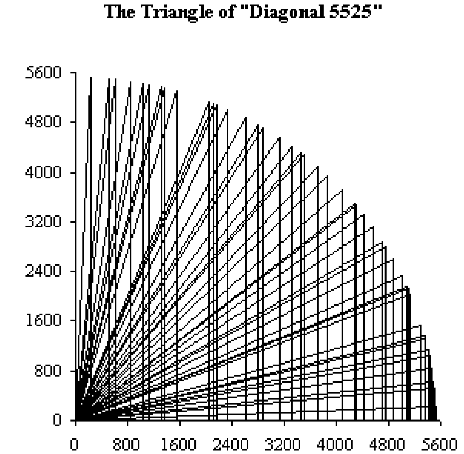
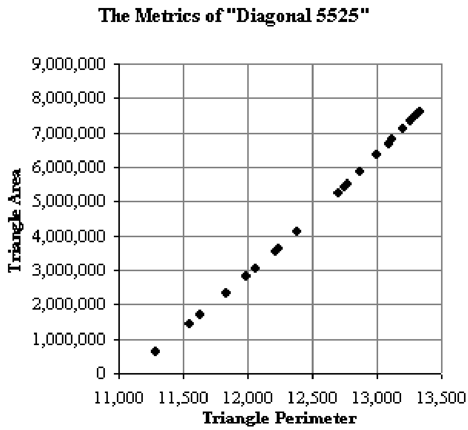
5,408	1,131	5,525	12,064	3,058,224
5,427	1,036	5,525	11,988	2,811,186
5,460	845	5,525	11,830	2,306,850
5,491	612	5,525	11,628	1,680,246
5,500	525	5,525	11,550	1,443,750
5,520	235	5,525	11,280	648,600





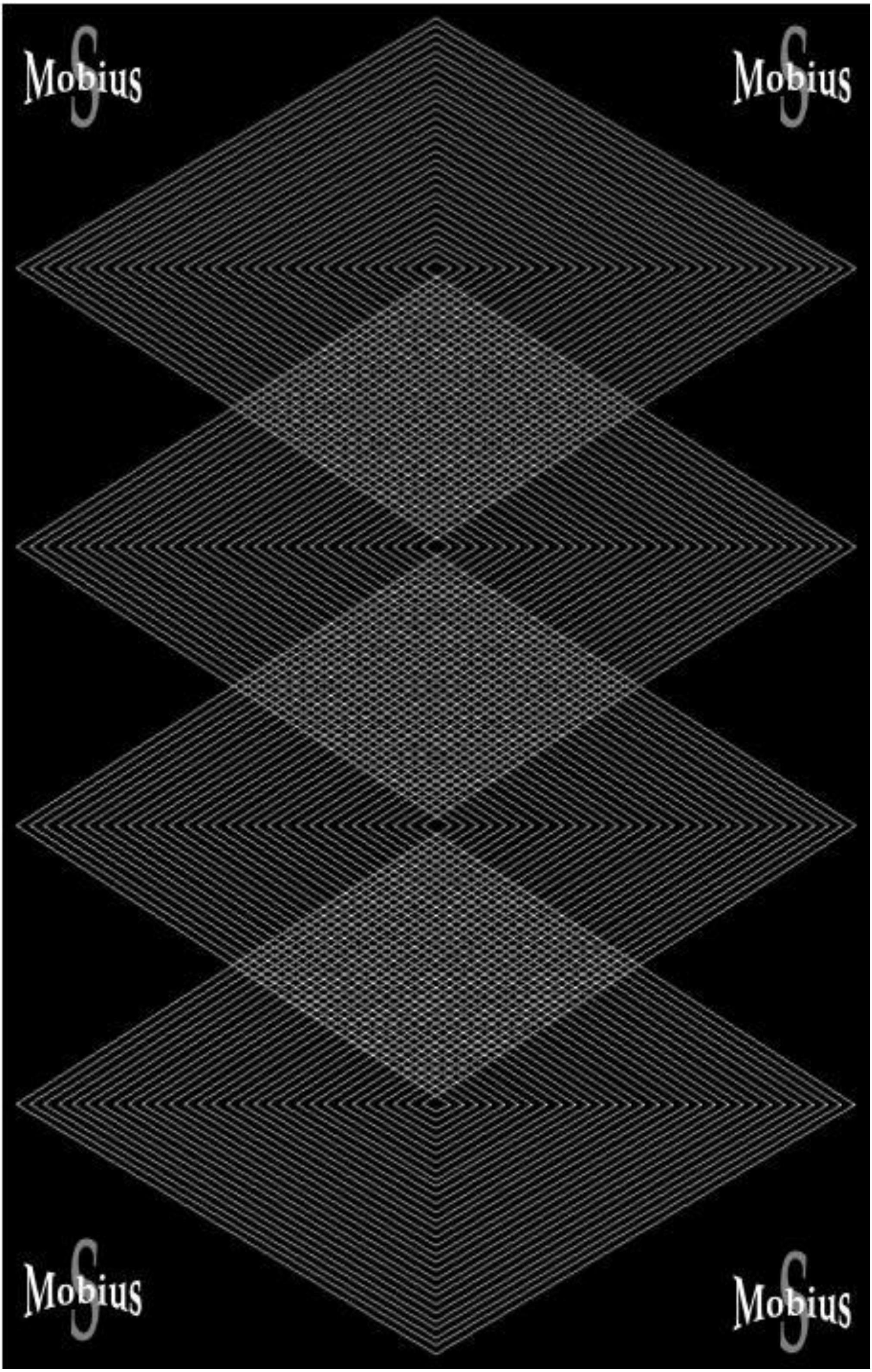
Our Second Wind

Once we caught our "Second Wind", it's clear the sky is the limit as far as what's possible. How many more questions come to mind – naturally? Is this a lesson plan in how to do all of this? Hardly. In fact, the joy in all of this is doing all of this. And in the process, what was necessary to do all of this "play"? A sampling:

Trigonometric Functions – sin, cos, arctan
Radian Measure
Distance Measurement
Angle Theorems
Euclidean Theorems
Programming
Plus a whole lot more!

But most important is the idea I could do all of this – by myself. And, if necessary, I could do it again. Right now. Starting with a blank spreadsheet.

And to what end? What good was any of this? What "practical application" is there here? As Faraday properly said, "What good is a newborn baby?" At this point, I'm just playing around. In the process, I've likely learned more about math than an introductory algebra/trip college class! Memorization? Hardly. And will there be bumps in the road? You bet. But this path of educational investigation truly is "the path less taken".


Mobius
Mobius
Mobius
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