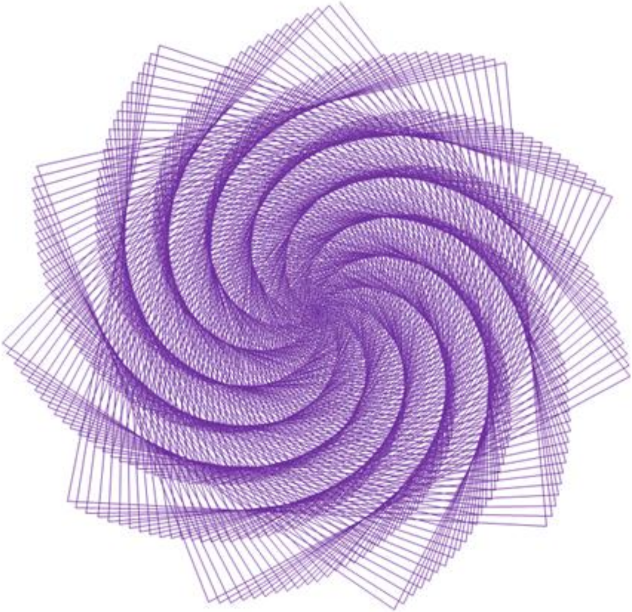
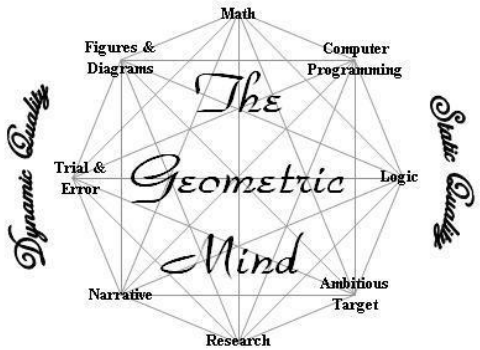
THE GEOMETRIC MIND SERIES
an *auto*SOCRATIC QUICK-START publication

The Spiral

Beauty - From Cartesian to Polar Coordinates

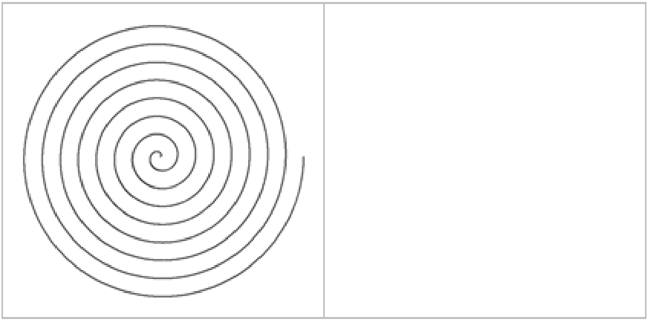




Published by:
*auto*SOCRATIC PRESS
www.rationalsys.com

MAKING A SPIRAL
AN INTRODUCTION

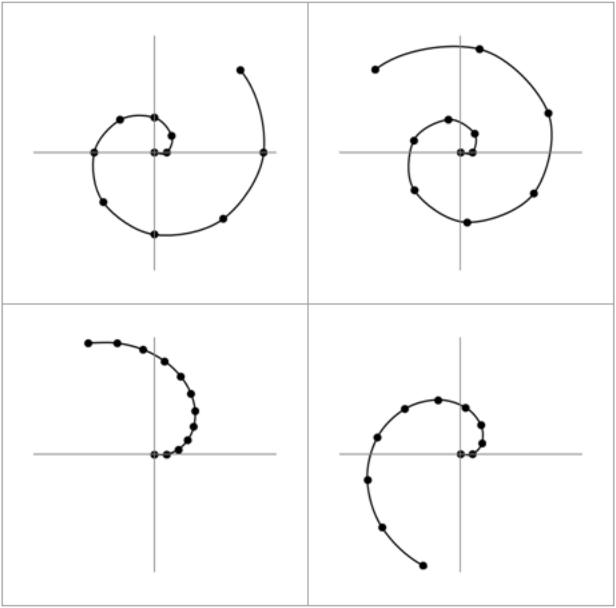
Spirals are beautiful – and easy to make. Re-create the spiral below to see for yourself:



It's easy to do. However, as you can see in the following examples, there are an infinite number of ways to do this!

MAKING A SPIRAL

There are an infinite number of ways!

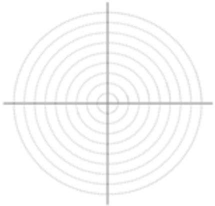


What would I need to tell someone else how I made my spiral?

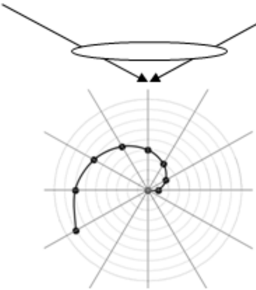
Making a Spiral

Two Requirements

DISTANCE
I need to know how far each point is from the center.

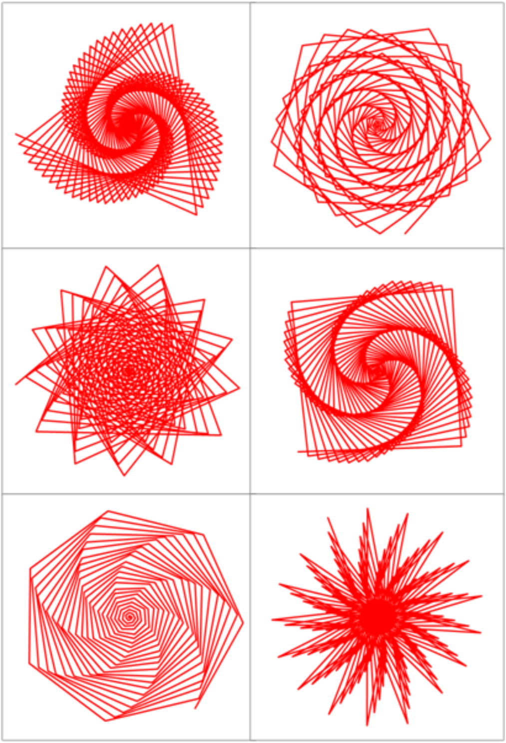


ANGLE
I need to know how far to move around the circle.



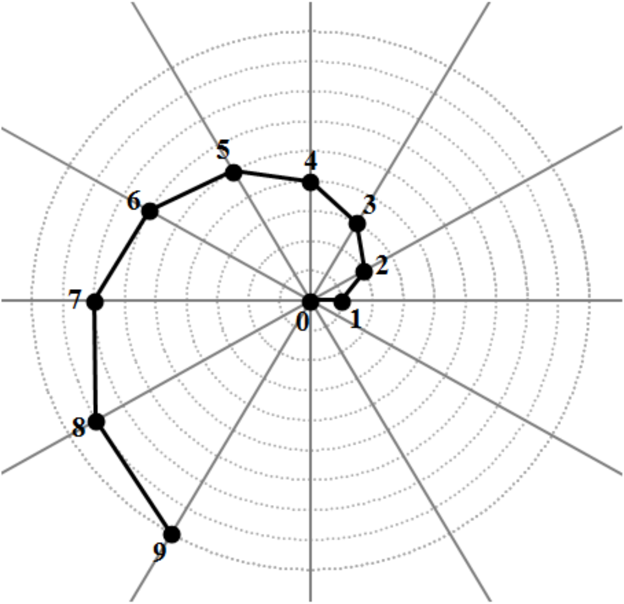
THE SPIRAL
I will be able to plot - and describe - my spiral.

MORE EXAMPLES



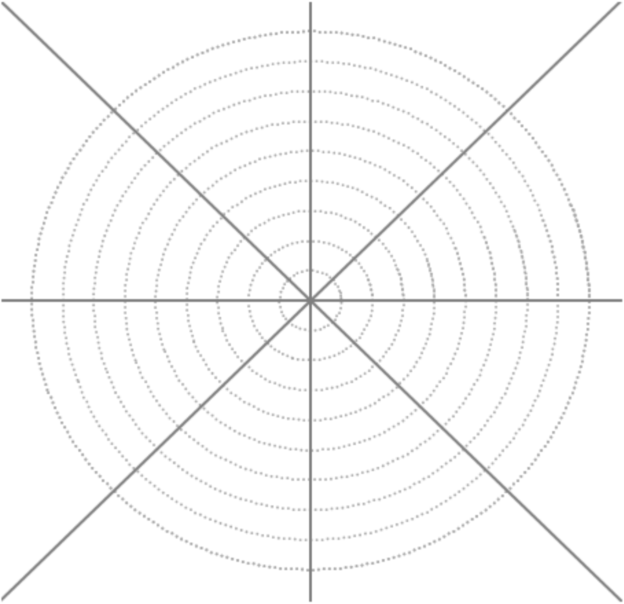
MAKE A SPIRAL

Plot 0-9, Skipping 30°



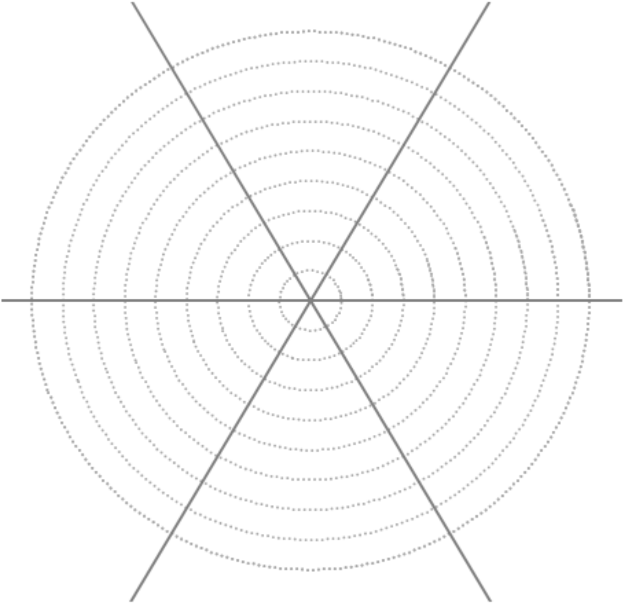
MAKE A SPIRAL

Plot 0-9, Skipping 45°



MAKE A SPIRAL

Plot 0-9, Skipping 60°

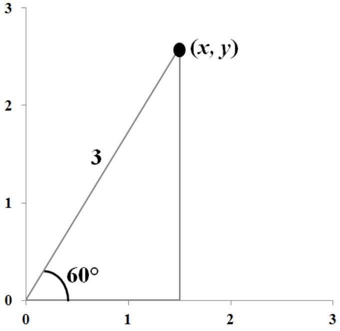


DESCRIBING A POINT'S LOCATION

Two Methods

In plotting points, I usually march over x units and up y units. Now, I'm turning a certain number of degrees and going up a diagonal. But since these two methods are describing the same point, there must be some relationship between the two methods.

Let's take one example and see. Suppose I was plotting 60° and a distance of 3:

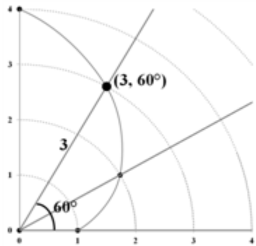


Describing a Point's Location

Two Methods

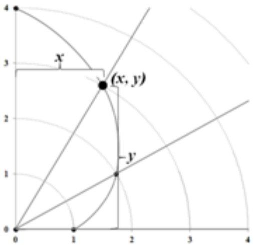
POLAR COORDINATES

I can describe this point by its angle and distance:



CARTESIAN COORDINATES

I can describe this point by its x and y distances:



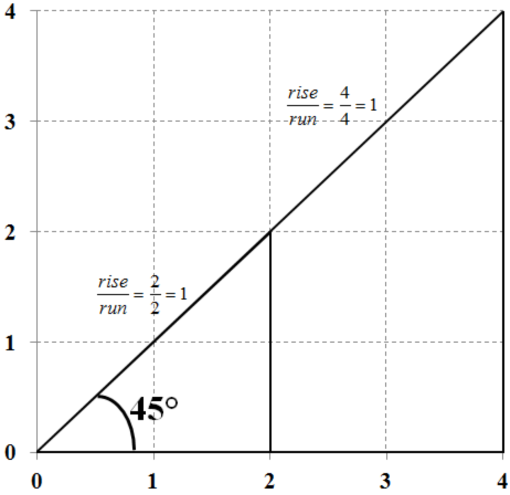
I can describe this point by both POLAR and CARTESIAN coordinates, there must be some relationship between the two:

$$(r, \theta) = (x, y)$$

DESCRIBING A POINT'S LOCATION

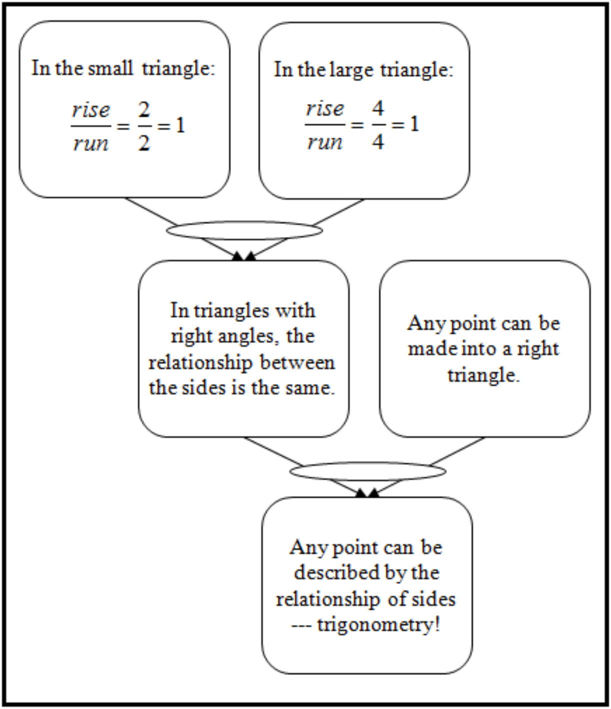
Two Methods

Fine. There are two methods: POLAR coordinates and CARTESIAN coordinates. How do I find each? Look at these two triangles:



Trigonometry

The Relationship of Sides in a Right Triangle.



Right Triangles

And Trigonometry

In a right triangle, there is a relationship between the three sides.

The trigonometric relationship is often remembered by:

SohCahToa

The trigonometry equations are:

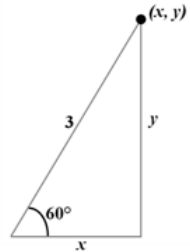
$$Sin(\theta) = \frac{opposite}{hypotenuse} \qquad Cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$Tan(\theta) = \frac{opposite}{adjacent}$$

Polar and Cartesian Coordinates

Solving for x and y

In a right triangle, there is a relationship between the three sides.



SohCahToa

$$Sin(\theta) = \frac{opposite}{hypotenuse}$$

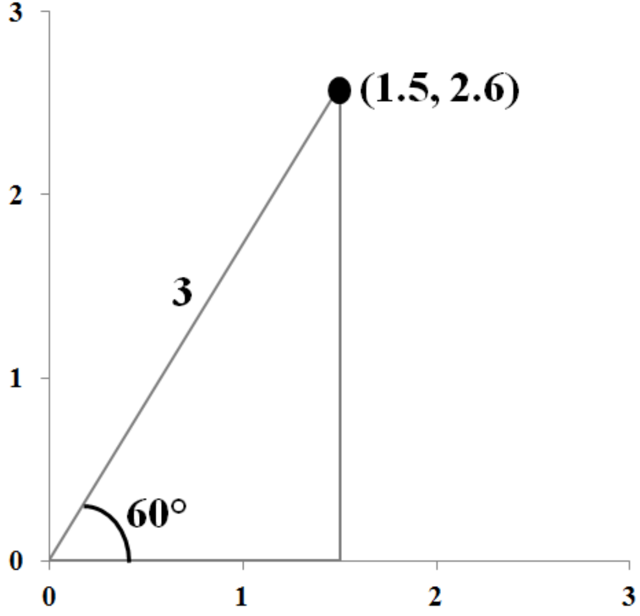
$$Cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$sin(60°) = \frac{y}{3} \qquad cos(60°) = \frac{x}{3}$$

$$3sin(60°) = y \qquad 3cos(60°) = x$$

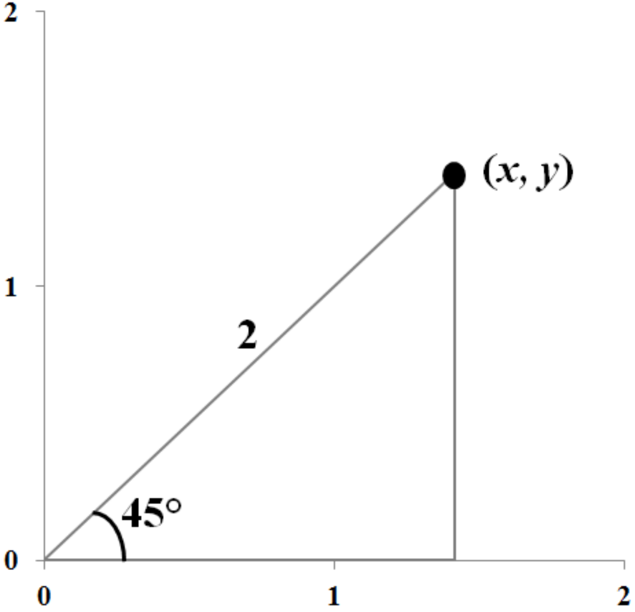
$$2.60 = y \qquad 1.50 = x$$

PUTTING IT ALL TOGETHER
(Making Sure We're Right, In Other Words)



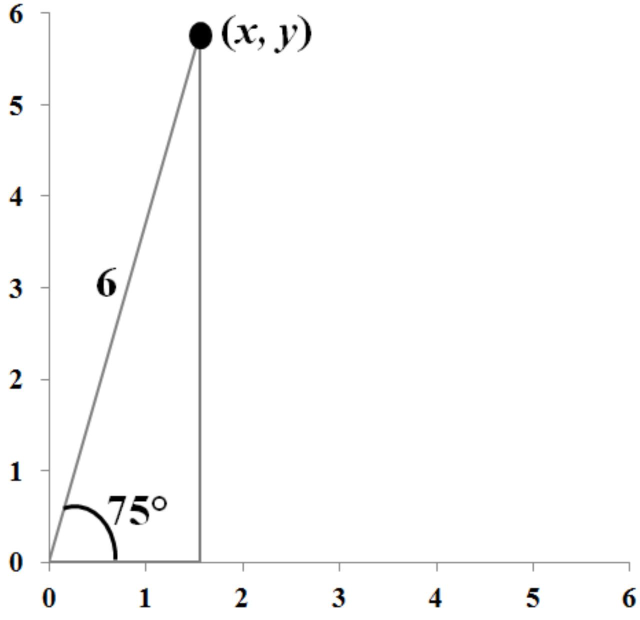
SAMPLE PROBLEM

Find (*x*. *y*)



SAMPLE PROBLEM

Find (*x*, *y*)



PROBLEMS

The following three problems each have a CHECK (to make sure you've done the problem right).

Once you've confirmed you've done the problem right, there's a KEY. The key is necessary to unlock the next installment.

Key1 Key2 Key3

PROBLEM 1

$$\cos(44°) = \frac{x}{3}$$

2	.		
Check		Key1	

PROBLEM 2

Number	Degree	*x*	*y*
0	0		
1	0		
2	40		
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			?

	1	.		
	Check		Key2	

PROBLEM 3

$$\left(r,\ 53.1^\circ\right) = \left(3,4\right)$$

	.		0
Key 3			Check

THE GEOMETRIC MIND

CONCEPT CARD

If you're talking about a triangle, draw the triangle. It's amazing how many times this simple step is ignored. It makes things so much easier!

TWO OTHER THINGS

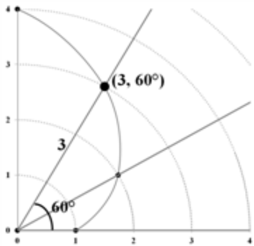
A. POLAR and CARTESIAN Coordinates
   Describing a Point's Location

B. Trigonometry
   The "measure of triangles", specifically, right triangles.

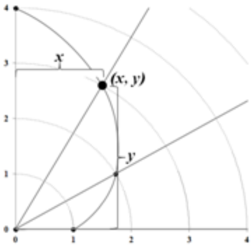
Describing a Point's Location

Two Methods

POLAR COORDINATES
I can describe this point by its angle and distance:



CARTESIAN COORDINATES
I can describe this point by its x and y distances:



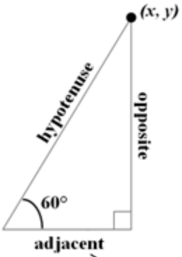
I can describe this point by both POLAR and CARTESIAN coordinates, there must be some relationship between the two:

$$(r,\theta)=(x,y)$$

Right Triangles

And Trigonometry

In a right triangle, there is a relationship between the three sides.



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The trigonometry equations are:

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